The Underemployment Trap*

Jie Duan[a], Paul Jackson[b,*]

[a] *Department of Economics, National University of Singapore, BLK AS1, #01-02, 1 Arts Link, Singapore 117570*

[b] *Department of Economics, National University of Singapore, BLK AS2, #04-22, 1 Arts Link, Singapore 117570*

Abstract

Many college graduates are underemployed, i.e., work in occupations that do not require a college degree. We document that underemployed workers are less likely to transition to a college occupation the longer they are underemployed and that longer underemployment histories are associated with lower wages in college occupations. To explain these findings, we develop a directed search model with unobserved heterogeneity, occupation-specific human capital, and on the job search. Workers are uncertain about their suitability for college jobs and learn through search. Underemployment is generated by search and information frictions, as workers with a low expected job-finding probability in college occupations self-select into underemployment. Once underemployed, workers' college occupation-specific human capital decays. A quantitative decomposition shows that unobserved heterogeneity explains most of the duration dependence in underemployment.

Keywords: Underemployment, Duration Dependence, Unobserved Heterogeneity, Human Capital

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*Corresponding author

Email addresses: duanjie@u.nus.edu (Jie Duan), ecspgj@nus.edu.sg (Paul Jackson)

1. Introduction

A significant fraction of college graduates in the US are underemployed, i.e., work in jobs that do not typically require a college degree. While underemployment is not a new phenomenon, it has gained considerable attention since the Great Recession with a growing consensus that underemployed graduates are trapped, unable to escape their low-wage jobs.[1] Despite its traction in the media, research which studies underemployment is still in its infancy. We know that recent graduates are nearly ten times more likely to be underemployed than unemployed and that the underemployment rate is countercyclical (Barnichon and Zylberberg, 2019).[2] However, little is known about the quintessential underemployment duration, whether underemployed graduates are indeed stuck and, if so, what the sources of the underemployment trap are.

This paper studies the features and determinants of underemployment durations by first reporting several new stylized facts. Most prominently, we document negative duration dependence in underemployment. That is, the longer a worker has been underemployed, the less likely they are to transition to an occupation that requires a college degree. We then develop a directed search model which generates duration dependence in underemployment. Finally, we decompose duration dependence into two classic channels: dynamic selection based on unobserved heterogeneity and structural duration dependence generated through the growth and decay of occupation-specific human capital.

We use the National Longitudinal Survey of Youth 1997 (NLSY97) to document three facts. First, the average underemployment duration is nearly eighteen months. Second, the probability an underemployed graduate transitions to a college occupation is decreasing in the length of their underemployment spell. For example, a worker who has been underemployed for a year is nearly 40% less likely to exit underemployment than a newly underemployed

[1] For a few examples, see "First jobs matter: Avoiding the underemployment trap" by Michelle Weise and "College Grads May Be Stuck in Low-Skill Jobs" by Ben Casselman.

[2] The underemployment rate is typically placed at nearly 40%. See Abel et al. (2014), BGT and SI (2018), Barnichon and Zylberberg (2019), and Jackson (2023).

worker. Third, an additional month of underemployment history is associated with 0.13% lower wages in college occupations.

To explain these facts, we develop a model of underemployment grounded in the environments of Gonzalez and Shi (2010) and Menzio and Shi (2011). Workers enter the labor market and direct their search towards non-college or college jobs. The first key ingredient is that workers can be of either limited- or broad-suitability, where a worker's type determines the probability they will produce output at any given college job. As in Gonzalez and Shi (2010), there is symmetric incomplete information regarding a worker's type and learning occurs through search. This captures the notion that recent graduates are uncertain about their best fit in the labor market, especially before they begin applying for jobs.[3] Those with a low expected suitability self-select into underemployment and continue to search on the job. It is at this stage where the model's second key ingredient kicks in: underemployment leads to the accumulation (decay) of non-college (college) occupation-specific human capital.

The model produces an optimality condition relating the marginal cost and benefit of exiting underemployment and encompasses the two channels which generate duration dependence. First, workers with a longer underemployment duration are more likely to be a limited suitability type and therefore less likely to match with a college job. This is the unobserved heterogeneity channel. Second, remaining underemployed makes workers more (less) productive in non-college (college) jobs, reducing the marginal benefit of exiting underemployment. This is the human capital dynamics channel. We make an identifying restriction to disentangle the contribution of each channel to duration dependence in underemployment by assuming that wages are independent of the worker's unobserved type. This assumption allows the human capital dynamics channel to be identified by matching the relationship between wages and underemployment history, leaving the unobserved heterogeneity channel

[3] Our emphasis on incomplete information is informed by Conlon et al. (2018) and Baley et al. (2022), who document empirical support for information frictions and learning, and related work studying the role of information frictions in shaping individual labor market outcomes (Papageorgiou, 2014; Gervais et al., 2016; Guvenen et al., 2020).

to be primarily pinned down by targeting the duration dependence profile.

With our identification strategy in hand, the model is calibrated and used to decompose the model generated duration dependence into the two aforementioned channels. The model can match well, among other moments, the path of transition probabilities between non-college and college jobs and the relationship between underemployment and wages. In our main exercise, we shut down the human capital dynamics channel and find that the model with only unobserved heterogeneity can explain 95.27% of the duration dependence observed in the data. When we shut down the unobserved heterogeneity channel, the model fails to produce underemployment. Moreover, the model does not generate enough duration dependence when a worker's type is observable. This underscores the role of unobserved heterogeneity in generating underemployment and the ensuing duration dependence.

We also assess the role of bad luck versus sorting in generating long underemployment durations. We find that even broad-suitable workers who take longer to find their first job, which can occur out of bad luck due to search frictions, do not experience significantly longer underemployment spells than their lucky peers. This again points to the role of sorting in generating duration dependence in underemployment, and implies that there is a weak relationship between a worker's unemployment and underemployment duration. We find that, just as in the model, there is a weak correlation between the length of unemployment and underemployment spells in the data.

Finally, we provide empirical backing for key modelling assumptions and assess the robustness of our quantitative results. In particular, our identifying restriction is supported by the fact that duration dependence in wages for transitions from underemployment is small, especially relative to transitions from unemployment. Therefore, our decomposition is not very responsive to allowing wages in college jobs to be correlated with a worker's type. On the contrary, given the high degree of duration dependence in wages in transitions from unemployment, a decomposition of duration dependence in unemployment would be much more sensitive under our identification strategy.

Our paper relates to the growing literature which studies underemployment. We are unaware of any study which has documented duration dependence in underemployment or the relation between underemployment and wages in college jobs. Many existing models generate underemployment in random search environments (e.g., Shephard and Sidibé (2022) and Jackson (2023)).[4] An exception is Barnichon and Zylberberg (2019), where workers direct their search to islands. We develop a competitive search model that generates duration dependence in underemployment, which is absent from the aforementioned models. Finally, we emphasize selection and information frictions as sources of underemployment and the resulting duration dependence.

Underemployment is related to skill mismatch (Guvenen et al., 2020; Lise and Postel-Vinay, 2020; Baley et al., 2022).[5] While these papers have developed innovative approaches for studying the implications of skill mismatch over the business- and life-cycle, we focus on educational mismatch for several reasons. First, the measurement of workers' skills in those papers do not account for skills acquired in college as they are based on test scores measured before most individuals attend college. Second, underemployment has garnered significant attention as many countries implement policies to increase the supply of college graduates. A more thorough understanding of the sources and properties of underemployment has the potential to contribute to such policy discussions.

We draw on the literature which has studied duration dependence in unemployment.[6] Our main findings are consistent with an emerging body of evidence showing that selection can account for a vast majority of duration dependence in unemployment (Jarosch and Pilossoph, 2019; Mueller et al., 2021; Alvarez et al., 2023). Our modelling of skill loss during underemployment is inspired by the literature on skill loss during unemployment (Pissarides, 1992; Ljungqvist and Sargent, 1998). We complement this literature by documenting the

[4] See Albrecht and Vroman (2002), Gautier (2002), Dolado et al. (2009), and Coskun (2020).

[5] We measure occupational skill requirements as in Guvenen et al. (2020) and show that they are positively correlated with education requirements. See Figures A.1-A.2.

[6] Recent references include Barnichon and Figura (2015), Doppelt (2016), Fernández-Blanco and Preugschat (2018), Kospentaris (2021), and Baydur and Xu (2024).

relation between underemployment and wages in college occupations and by modelling the loss of occupation-specific human capital during underemployment.

The rest of this paper is organized as follows. Section 2 presents the empirical evidence. Section 3 introduces the model, while Section 4 defines a stationary equilibrium and characterizes the sources of duration dependence. Section 5 presents the quantitative analysis and Section 6 concludes. Appendices A-E are in the online supplementary materials.

2. Empirical Evidence

This section documents three facts: (i) underemployment is more prevalent and persistent than unemployment, (ii) underemployment exhibits negative duration dependence, and (iii) longer underemployment histories are associated with lower wages in college jobs.

We use the NLSY97 and Occupational Informational Network (O*NET).[7] From the NLSY97, we construct a weekly history of graduates from when they enter the labor market until 2011. An individual's history begins when they graduate with a BA or above and are not enrolled in college thereafter. We arrive at a sample of 996 who obtained a BA or above before 2011 and have a complete set of time-varying individual characteristics.

An individual with a BA or above is underemployed (properly employed) if they work in a non-college (college) occupation. Following Abel et al. (2014) and Jackson (2023), non-college (college) occupations are those where less than (at least) 50% of respondents in O*NET releases 5.0-16.0 state that a BA or above is necessary to perform that occupation.[8]

Measuring occupational mobility is prone to measurement error (Moscarini and Thomsson, 2007). While this concern is mitigated in our analysis because we focus on transitions between two broad groups of occupations, we attempt to identify "genuine" switches. Transi-

[7] Descriptions of both surveys are in Appendices A.1-A.2.

[8] Table A5 lists occupations around the 50% threshold while the ten most common college and non-college occupations are listed in Table A6. The 50% cutoff produces an underemployment rate that is similar to alternative measures of education requirements (BGT and SI, 2018; Barnichon and Zylberberg, 2019). Appendix B assesses the robustness of our findings to alternative measures of educational requirements.

tions from a non-college to college occupation that are accompanied by a change in employer are treated as genuine. For within-firm switches, we use a three-step correction. First, we measure skill requirements following Guvenen et al. (2020), producing a skill requirement vector, $\mathbf{r}_i$, for each occupation i. Second, we compute the angular distance between two occupations, $\phi(\mathbf{r}_i, \mathbf{r}_j)$, when a worker transitions between occupation i and j. The final step is to label the transition as genuine if $\phi(\mathbf{r}_i, \mathbf{r}_j) \geqslant \bar{\phi}$, i.e., the occupations have sufficiently different skill requirements. The threshold, $\bar{\phi}$, is chosen so that the correlation in skill requirements between occupations is close to zero.[9]

2.1. The Prevalence and Persistence of Underemployment

To highlight the prevalence of underemployment, we calculate the fraction of a respondent's history spent in each labor force status. From the first row of Table 1, respondents spent 39.2% of their post-graduate career underemployed. The second row of Table 1 shows that the average underemployment duration is around 18 months.

[INSERT TABLE 1 AROUND HERE.]

2.2. Duration Dependence

We define exiting underemployment to be when a worker transitions from a non-college to a college occupation between week $t-1$ and t.[10] Our objective is to estimate the negative exponential relationship between the probability of transitioning from underemployment to proper employment and the worker's underemployment duration.[11] Specifically, we estimate

[9] Appendices A.3-A.4 provide more details on this correction.

[10] The duration dependence is similar if we allow for three weeks between transitions. See Figure A.5(a).

[11] This approach follows Jarosch and Pilossoph (2019). Workers with an underemployment duration $\geqslant 24$ months are grouped together, as there are rarely transitions from underemployment at such durations. Figure A.5(b) shows that the results are largely unchanged under a maximum duration of 30 months.

the following via weighted nonlinear least squares:

$$D(\tau) = b_1 + \ (1 - b_1)\exp(-b_2 \times \tau), \tag{1}$$

where $D(\tau)$ is the average exit probability at duration τ relative to the average exit probability of workers who have been underemployed for less than one month. To estimate (1), we need estimates of the average exit probabilities at each duration τ. These are obtained by estimating

$$y_{it} = \beta\tau_{it} + \Gamma \cdot X_{it} + \delta_t + \epsilon_{it}, \tag{2}$$

where y_{it} is an indicator for whether individual i exited underemployment during period t, τ is underemployment duration, and δ_t contains month and year fixed effects. The vector X contains gender, race, age, gender interacted with race, ASVAB quartile, family income, outstanding student loan debt, highest degree, gender interacted with highest degree, undergraduate GPA, undergraduate major (STEM or Arts and Social Sciences), and job satisfaction.[12] We then compute the predicted transition probabilities at each duration $\tau \in \{1, \ldots, 24\}$ relative to $\tau = 0$.

Figure 1 displays the results. The triangles and circles represent the predicted transition probabilities generated by equation (2) with and without individual level controls, respectively. The curves are the result of estimating (1) on each set of relative transition probabilities. Controlling for observable characteristics considerably attenuates the duration dependence profile. While we find that demographics and ASVAB score have a sizeable effect on the duration dependence profile, the worker's job satisfaction has the largest impact on attenuating the profile.[13] After controlling for observable characteristics, we still observe a decline in the relative transition probability over the first year of underemployment, before leveling off at higher underemployment durations.

[12]Figure A.4 shows that job satisfaction is significantly higher in college occupations. Table A7 shows that STEM (Arts and Social Sciences) majors spent 32.5% (42.9%) of their labor market history underemployed.

[13]See Figure A.3 for a detailed analysis on the attenuation of the duration dependence profile.

The shape of the decline in the relative exit probability is indicative of selection. If workers, based on unobservable characteristics, have different propensities to exit underemployment, then workers with a high exit probability will quickly leave while the long term underemployed will be primarily comprised of those with a low exit probability. There could also be structural forces causing each individual's probability of exiting underemployment to decline. To examine if there is evidence supporting this, we proceed to study the relationship between underemployment and wages.

[INSERT FIGURE 1 AROUND HERE.]

2.3. Wages and Underemployment

Longer unemployment durations are associated with lower wages (Ortego-Marti, 2016; Laureys, 2021). This fact is consistent with two prominent explanations for structural duration dependence: human capital depreciation and statistical discrimination. In the former, a worker's skills depreciate over their unemployment spell, which lowers their productivity and chance to find a job. In the latter, a longer unemployment duration signals that the worker is less productive. To assess whether workers with a longer underemployment history earn lower wages in college jobs, we estimate

$$w_{it} = \alpha \text{Underhis}_{it} + \beta \text{College}_{it} + \mu \text{Underhis}_{it} \times \text{College}_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \qquad (3)$$

where w_{it} is individual i's log wage in period t, Underhis is accumulated experience in non-college occupations, and College is a dummy for being employed in a college occupation. The vector X includes a cubic in potential experience, annual regional and national unemployment rate, a quadratic in age, family income, student loan debt, job satisfaction, region, and two-digit industry fixed effects. Finally, δ_i is an individual fixed effect.

Table 2 presents the results. Column (6) is our preferred specification and shows that an additional month of underemployment is associated with 0.06% higher wages in non-college jobs and 0.13% lower wages in college occupations.[14] Moreover, we find that an additional month of unemployment (Unhis) is associated with a 1.36-1.45% decline in wages, which is consistent with prior literature (Addison and Portugal, 1989; Neal, 1995).

[INSERT TABLE 2 AROUND HERE.]

2.4. From Empirics to Theory

This section gives a preview of how the model will generate duration dependence in underemployment and the results of Table 2. As mentioned above, the patterns in Figure 1 are indicative of selection. To allow for this, we consider workers who are heterogeneous in their (unobservable) suitability for college jobs. Workers with a high (low) suitability for college jobs quickly (slowly) exit underemployment. Thus, the exit rate will decrease over the underemployment spell as the composition of workers shifts towards those with limited suitability.

To reconcile the findings in Table 2, we assume underemployed workers accumulate non-college occupation-specific human capital, making them more productive in non-college jobs.[15] At the same time, workers do not utilize their college-specific skills while underemployed. Therefore, a worker who transitions to a college job after a long underemployment spell will be less productive than one who quickly left underemployment. The former effect leads to higher wages in non-college jobs, while the latter reduces wages in college jobs. Occupation-specific human capital dynamics also generates duration dependence, as

[14] We conduct several robustness exercises. First, we use one-digit industry and occupation fixed effects (Table A8). Second, we control for one-digit or two-digit occupation fixed effects in all specifications to address the concern of occupation heterogeneity (Altonji and Shakotko, 1987) (Tables A9 and A10). Finally, we include month and year fixed effects (Table A11). The results are similar across all specifications.

[15] Our modelling of human capital aligns with evidence that human capital is occupation-specific (Kambourov and Manovskii, 2009). To further support this approach, Table A4 shows that the association between underemployment and wages is decreasing in the distance in skill requirements between non-college and college occupations. The idea here is that with a larger distance in skill requirements, skills required by the college occupation would not have been used as intensively while underemployed, causing skills used by the college occupation to decay at a greater rate and larger wage losses.

the benefit of exiting underemployment decreases as workers become more (less) productive in non-college (college) jobs.

To identify how much of the duration dependence in underemployment is driven by selection and human capital dynamics, we impose an identifying restriction whereby duration dependence in wages is completely driven by human capital dynamics and not the worker's suitability type. We employ this identification strategy as there is little duration dependence in wages for transitions from underemployment. For context, column (6) of Table 2 shows that an additional month of underemployment (unemployment) is associated with a 0.13% (1.4%) decrease in college wages. As such, our decomposition of duration dependence in underemployment does not change much when we allow wages to be correlated with a worker's type. However, a decomposition of duration dependence in unemployment would be much more sensitive under this identification strategy, as there is significantly more duration dependence in wages for transitions from unemployment than underemployment. Sections 5.4-5.5 present additional evidence supporting the identifying restriction and robustness checks.

3. Model

Time is discrete and goes on forever. There is a measure one of workers and a large measure of firms. All agents are risk neutral and share the discount factor $\beta \in (0,1)$. Firms are indexed by $\chi \in X = \{n, c\}$, where n (c) denotes a non-college (college) job.

Workers are ex-ante heterogeneous in their suitability type $i \in \{L, H\}$ for college jobs, where the mass of type H workers is $\pi \in (0,1)$. We refer to type H (L) workers as broad- (limited-) suitable. A type i worker is suitable for any given college job with probability a^i, where $a^H > a^L$. Workers produce zero output in college jobs they are unsuitable for. There is symmetric incomplete information regarding a worker's suitability type.

Workers are also heterogeneous in their labor market history, where $\upsilon \in \Upsilon = \{0, 1, \ldots, \bar{\upsilon}\}$ is the number of periods a worker has been unemployed and $\tau \in T = \{0, 1, \ldots, \bar{\tau}\}$ is the worker's underemployment history (experience in non-college jobs). A worker's history, (υ, τ), is public information.

Upon meeting a worker, firms with college jobs observe a private signal which perfectly identifies unsuitable workers, allowing them to hire suitable workers and reject unsuitable ones. Neither workers nor other firms observe the signal. Additionally, firms operate a technology that maps one unit of suitable labor into $y_\chi(\tau)$ units of output where $y_\chi \colon T \to \mathbb{R}_+$, y_n (y_c) is weakly increasing (decreasing) in τ, and $y_c(\tau) > y_n(\tau)$ for all $\tau \in T$.

The labor market is organized in a continuum of submarkets indexed by $\omega = (\chi, \upsilon, \tau, x)$. In submarket ω, type χ firms search for workers with history (υ, τ) and offer suitable workers a contract worth $x \in \mathbb{R}$ in lifetime utility.

Each period is divided into four stages: search, matching, entry/production, and exit. In stage 1, firms incur a cost k_χ to post a type χ vacancy and workers select a submarket to search in. Employed workers are endowed with $\lambda \in [0, 1]$ units of search intensity.

In stage 2, suitable workers and vacancies search for each other. Let $v(\omega)$ denote the measure of vacancies in submarket ω. Further, $u^i(\omega)$ and $e^i(\omega)$ denote the measure of unemployed and employed workers, respectively, of suitability type i searching in submarket ω. The effective measure of suitable workers is $\psi(\omega) = \sum_i a^i(\omega)[u^i(\omega) + \lambda e^i(\omega)]$, where $a^i(\omega) = 1\,(a^i)$ if $\chi = n\,(c)$. The number of matches is given by a constant returns to scale matching technology $F(\psi(\omega), v(\omega))$. Defining $\theta(\omega) \equiv v(\omega)/\psi(\omega)$ as tightness, suitable unemployed workers match with probability $p(\theta) = F/\psi$, where $p(\theta)$ is strictly increasing and concave, $p(0) = 0$, and $p(\infty) = 1$. Suitable workers searching on the job find a match with probability $\lambda p(\theta)$. Vacancies are filled with probability $q(\theta(\omega)) = F/v$, where $q(\theta)$ is strictly decreasing and convex, $q(0) = 1$, and $q(\infty) = 0$.

In stage 3, a measure δ of workers enter the labor market unemployed. Matches (unemployed workers) produce $y_\chi(\tau)$ (b) units of output. After producing at a college job, workers with $\tau > 0$ regain their skills with probability ϕ. Workers exit the market with probability δ in stage 4. There are no transitions from employment to unemployment.

Let μ denote the worker's expectation that they will produce output in a college job. Their initial belief is $\mu_0 = \pi a^H + (1-\pi)a^L$. Unemployed workers who search for and do not find a college job update their beliefs, using Bayes rule, to

$$\hat{\mu} \equiv H(p,\mu) = a^H - \frac{(a^H - \mu)(1 - pa^L)}{1 - p\mu}, \tag{4}$$

where $p = p(\theta)$. Underemployed workers update their beliefs to $H(\lambda p, \mu)$. Equation (4) also captures firms' beliefs about a worker's suitability, as there is symmetric incomplete information. Moreover, (υ, τ) is a sufficient statistic for μ as (υ, τ) captures how many times the worker's expected suitability was updated according to (4).[16]

The contract space is complete, which ensures that contracts offered by firms are bilaterally efficient (Menzio and Shi, 2009, 2011). As multiple contracts can deliver bilateral efficiency, and wage data is used to estimate the model, we are explicit about the employment contract. We follow Schaal (2017) and Baley et al. (2022) by assuming firms offer a contract where wages equal match output and workers incur a one-time hiring fee to compensate firms for their recruitment costs. Under this contract, firms earn a share of the match surplus through the one-time hiring fee, while the worker's share is the match surplus net of the hiring fee.

4. Equilibrium

As employment contracts are bilaterally efficient, it is without loss of generality to solve the model where submarkets are indexed by the value delivered to the worker, x, and by characterizing the joint value of a match. Let $V_{u,\chi}(\upsilon, \mu)$ denote the lifetime utility of an unemployed worker, measured at the beginning of stage 3, with unemployment history υ, expected suitability μ, and who searches in a submarket with type χ jobs. They produce b and remain in

[16] We show in Proposition 1 that employment in college jobs is an absorbing state. Therefore, a worker with history (υ, τ) was unemployed for υ periods before becoming underemployed for τ periods.

the labor market with probability $1-\delta$. Suppose, in the next period, that the worker searches for a non-college job in submarket $\omega = (n, \hat{\upsilon}, 0, x)$ where $\hat{\upsilon} = \min\{\upsilon + 1, \bar{\upsilon}\}$. The worker finds a match and earns the continuation value, x, with probability $p(\theta(\omega))$. If the worker does not find a match, their continuation value is $V_u(\hat{\upsilon}, \mu) = \max\{V_{u,n}(\hat{\upsilon}, \mu), V_{u,c}(\hat{\upsilon}, \mu)\}$. It follows that $V_{u,n}(\upsilon, \mu)$ satisfies

$$V_{u,n}(\upsilon, \mu) = b + \beta(1-\delta)\{V_u(\hat{\upsilon}, \mu) + \max_x p(\theta(n, \hat{\upsilon}, 0, x))(x - V_u(\hat{\upsilon}, \mu))\}. \tag{5}$$

Now suppose that an unemployed worker searches for a college job. Workers expect to find a job with probability $\mu p(\theta(\omega))$. Those who do not find one update their beliefs to $\hat{\mu} = H(p(\theta(\omega)), \mu)$. The value of searching for a college job satisfies:

$$V_{u,c}(\upsilon, \mu) = b + \beta(1-\delta)\{V_u(\hat{\upsilon}, \hat{\mu}) + \mu \max_x p(\theta(c, \hat{\upsilon}, 0, x))(x - V_u(\hat{\upsilon}, \hat{\mu}))\}. \tag{6}$$

Let $V_{e,n}(\upsilon, \tau, \mu)$ denote the joint value of a match between a non-college job and a worker with characteristics (υ, τ, μ). The match produces $y_n(\tau)$ units of output. If the worker remains in the market, they transition to another job with probability $\lambda\mu p(\theta(\omega))$ and the worker's (firm's) continuation value is x (0). If the match survives, the continuation value is $V_{e,n}(\upsilon, \hat{\tau}, \hat{\mu})$, where $\hat{\tau} = \min\{\tau + 1, \bar{\tau}\}$. Thus, $V_{e,n}(\upsilon, \tau, \mu)$ satisfies

$$V_{e,n}(\upsilon, \tau, \mu) = y_n(\tau) + \beta(1-\delta)\{V_{e,n}(\upsilon, \hat{\tau}, \hat{\mu}) + \lambda\mu \max_x p(\theta(c, \upsilon, \hat{\tau}, x))(x - V_{e,n}(\upsilon, \hat{\tau}, \hat{\mu}))\}. \tag{7}$$

Equation (7) implies that underemployed workers only search for college jobs. As shown in Proposition 1 in the supplementary materials, this is because workers employed in type χ jobs cannot generate additional surplus by transitioning to another type χ job.

For a worker with characteristics (υ, τ, μ) employed at a college job, the match produces $y_c(\tau)$ units of output. With probability ϕ, the worker regains their skills and produces $y_c(0)$

for the remainder of the match. The joint value, $V_{e,c}(\upsilon,\tau,\mu)$, satisfies

$$V_{e,c}(\upsilon,\tau,\mu)=y_c(\tau)+\beta(1-\delta)\{\phi V_{e,c}(\upsilon,0,\mu)+(1-\phi)V_{e,c}(\upsilon,\tau,\mu)\}. \tag{8}$$

Proposition 1 also shows that proper employment is an absorbing state, as properly employed workers cannot generate additional surplus by moving to a non-college job.

The firm's cost to create a vacancy is k_χ whereas the benefit from posting a vacancy in submarket $\omega=(\chi,\upsilon,\tau,x)$ is $q(\theta(\chi,\upsilon,\tau,x))\{V_{e,\chi}(\upsilon,\tau)-x\}$.[17] In submarkets visited by workers, tightness is consistent with firms' incentives to create vacancies if and only if

$$k_\chi \geqslant q(\theta(\chi,\upsilon,\tau,x))\{V_{e,\chi}(\upsilon,\tau)-x\}, \tag{9}$$

and $\theta(\chi,\upsilon,\tau,x)\geqslant 0$ with complementary slackness.

Definition 1. A stationary recursive equilibrium consists of a belief function $\hat{\mu}(p,\mu)$, tightness function $\theta(\omega)$, value and policy function for unemployed workers, $V_u(\upsilon,\mu)$ and $\omega_u^*(\upsilon,\mu)$, joint value and policy function, $V_{e,\chi}(\upsilon,\tau,\mu)$ and $\omega_{e,\chi}^*(\upsilon,\tau,\mu)$, and a distribution of workers that satisfies the following conditions. First, $\hat{\mu}(p,\mu)$ is given by (4). Second, $\theta(\omega)$ satisfies (9) and the slackness condition for all ω. Third, $V_u(\upsilon,\mu)=\max\{V_{u,n}(\upsilon,\mu),V_{u,c}(\upsilon,\mu)\}$ where $V_{u,n}(\upsilon,\mu)$ and $V_{u,c}(\upsilon,\mu)$ satisfy (5)-(6) and $\omega_u^*(\upsilon,\mu)$ is the associated policy function. Fourth, $V_{e,\chi}(\upsilon,\tau,\mu)$ for $\chi\in X$ satisfies (7)-(8) and $\omega_{e,\chi}^*(\upsilon,\tau,\mu)$ are the associated policy functions. The distribution of workers satisfies the laws of motion specified in Appendix D.1.

As established by Menzio and Shi (2010, 2011) for directed search models with free entry and bilateral efficiency and Schaal (2017) for similar environments with two-sided heterogeneity, a recursive equilibrium exists and is block-recursive (BRE). As workers self-select into submarkets based on their observable characteristics, firms know they will only meet one type of worker in their respective submarket. Additionally, the hiring protocol and

[17] $V_{e,\chi}(\upsilon,\tau)$ and $V_{e,\chi}(\upsilon,\tau,\mu)$ are equivalent as (υ,τ) is a sufficient statistic for μ.

matching function specification of Gonzalez and Shi (2010) implies that firms do not need to keep track of the composition of suitable workers in each submarket. Hence, tightness in each submarket is independent of the distribution of workers across employment statuses and the composition of worker suitability.

4.1. Duration Dependence in Underemployment

This subsection describes how the model generates duration dependence in underemployment and distinguishes between the underlying channels.[18] To begin, we combine the entry condition (9) with (7), which allows us to write the problem of an underemployed worker as a choice of tightness. The first order condition is given by

$$k_c \geqslant p'(\theta^*_{c,\upsilon,\tau})(V_{e,c}(\upsilon,\tau,\mu) - V_{e,n}(\upsilon,\tau,\hat{\mu})), \tag{10}$$

and $\theta^*_{c,\upsilon,\tau} \geqslant 0$ with complementary slackness. As workers exit underemployment with probability $\mu\lambda p(\theta^*_{c,\upsilon,\tau})$, there are two channels which generate duration dependence in underemployment. First, workers with higher τ have a lower expected suitability, μ. This is the unobserved heterogeneity channel. Second, underemployment reduces (increases) the worker's productivity in college (non-college) jobs, putting downward pressure on the surplus generated by escaping underemployment, tightness $\theta^*_{c,\upsilon,\tau}$, and the matching probability $p(\theta^*_{c,\upsilon,\tau})$. This is the human capital dynamics channel.

How can the model distinguish between unobserved heterogeneity and human capital dynamics? Recall that workers earn a wage that is equal to match output. Moreover, match output, $y_\chi(\tau)$, is independent of a worker's suitability type. This is our identifying restriction, as it means that the model's relationships between underemployment history and wages are completely driven by the growth and decay of occupation-specific human capital.

There is an additional outcome that can distinguish between the two channels. Con-

[18]We thank an anonymous referee whose suggestions led to the development of this subsection.

sider the path of transition probabilities from underemployment to proper employment $\{\mu_\tau \lambda p(\theta^*_{c,\upsilon,\tau})\}_{\tau=1}^{\bar{\tau}}$. Proposition 2 in the supplementary materials shows that the model with only human capital dynamics generates a transition path that is generally decreasing and concave, especially at higher values of τ. The intuition is that as τ increases, the surplus generated by a worker exiting underemployment decreases. As the surplus continues to decline, each additional period of underemployment causes a larger relative change in the match surplus, meaning that the responses of vacancies, tightness, and the exit probability become progressively larger as the worker remains underemployed.

With unobserved heterogeneity only, the path is generally decreasing and convex. This is because the path's shape is primarily determined by the evolution of the expected suitability, μ_τ, which typically has a convex shape (see Proposition 3 in the supplementary materials).[19] Intuitively, workers learn more from not finding a college job early in their underemployment spell, causing them to rapidly downgrade their beliefs. As the worker remains underemployed, the changes in μ_τ become progressively smaller, leading to a transition path that becomes flat at higher values of τ.

5. Quantitative Analysis

This section presents our calibration strategy, quantitative findings, evidence supporting key modelling assumptions, and robustness checks.

5.1. Calibration

A unit of time is one month, $\bar{\upsilon} = 12$, and $\bar{\tau} = 24$. The matching technology is $F(\psi, v) = \frac{\psi v}{\psi + v}$ and the production functions are $y_\chi(\upsilon, \tau) = g_\chi e^{(d_{\chi,\upsilon}(\upsilon - 1) + d_{\chi,\tau}\tau)}$ for $\chi \in X$, which allows for skill loss during unemployment. There are 16 parameters. The discount factor is $\beta = 0.95^{1/12}$

[19] The matching probability, $p(\theta)$, does not vary much with τ in our quantitative analysis as there are offsetting effects of an increase in τ on the surplus generated by exiting underemployment.

and we normalize the economy by setting $g_c = 1$. The remaining 14 parameters are calibrated via method of simulated moments (MSM) to match 33 moments. The first moment is $b/[\text{Average labor productivity}] = 0.71$ (Hall and Milgrom, 2008). Second is the college job wage premium, i.e., how much higher a worker's wage is in a college job than in a non-college job. Following the approach of Barnichon and Zylberberg (2019) gives a premium of 25.97%.[20] We also target the regression coefficients in column (6) of Table 2. The remaining moments are an unemployment rate of 8.1%,[21] underemployment rate of 41.6%, average months spent unemployed before entering underemployment for the first time (2.147), and the path of 24 transition probabilities from non-college to college jobs that controls for observable characteristics (the red line in Figure 1).

The skill parameters $\{d_{\chi,v}, d_{\chi,\tau}\}$ for $\chi \in X$ are chosen to match the regression coefficients in column (6) of Table 2. While each of the 10 remaining parameters impacts all of the remaining moments, one can view b as targeting the $b/[\text{Average labor productivity}]$ ratio, g_n the college job wage premium, and $\{k_n, k_c\}$ the unemployment and underemployment rates as the entry costs affect both the amount and composition of vacancies.

The unobserved heterogeneity parameters, $\{\pi, a^L, a^H\}$, are chosen to match two features of the path of transition probabilities and the average number of months spent unemployed before becoming underemployed for the first time. The lower bound of the transition path is informative of a^L. The reason is that $\mu \to a^L$ as $\tau \to \bar{\tau}$. In words, workers who remain underemployed learn that they are likely a limited-suitable worker. It follows that the transition path at higher underemployment durations is largely determined by a^L as $\lambda\mu_\tau p(\theta_\tau) \to \lambda a^L p(\theta_\tau)$ as $\tau \to \bar{\tau}$. Figure 2(a) shows that the transition path levels off at a higher (lower) transition probability with a higher (lower) value of a^L.

[INSERT FIGURE 2 AROUND HERE.]

[20] See Appendix C.1 for more details.

[21] We target the nonemployment rate because, in the data, there are a significant number of transitions from not in the labor force to employment.

The "convexity" of the transition path is informative for a^H. This is because, as a^H increases, broad-suitable workers leave underemployment at a higher rate. So, workers who remain underemployed quickly realize they are likely a limited-suitable worker. Figure 2(b) shows that as a^H increases, the transition path declines at a higher rate initially, as workers quickly downgrade their expected suitability, before leveling off and giving a transition path with a more pronounced convex shape.

We then utilize the occupation choice of an unemployed worker to calibrate π. Recall that a worker's initial expected suitability is $\mu_0 = \pi a^H + (1 - \pi)a^L$. Initial beliefs are important in determining which type of job an unemployed worker searches for. If the worker has lower initial beliefs, they will spend more time searching for a non-college job as there is little expected benefit to searching for a college job. With a^L and a^H pinned down by the lower bound and convexity of the transition path, π is the only free parameter in determining the initial beliefs. We calibrate π by targeting the average number of months between a worker entering the labor market and beginning their first underemployment spell, which we refer to as the U2N duration. Figure 2(c) displays the type of job workers search for at each unemployment duration. For example, at $\pi = 0.05$, the worker searches for a non-college job for their first five periods of unemployment, and then uses the remainder of their unemployment spell to search for a college job. The U2N duration associated with each value of π is listed next to the black arrows. As π decreases, workers search longer for a non-college job, causing the U2N duration to increase.

There are three remaining parameters $\{\lambda, \delta, \phi\}$ that can be interpreted as being chosen to fine-tune the model's fit to the 33 moments. Intuitively, λ directly affects the job-to-job transition probability of suitable workers, $\lambda p(\theta)$, while δ impacts the expected duration of a match and therefore tightness, θ. Therefore, adjusting $\{\lambda, \delta\}$ improves the model's fit to the path of transition probabilities. As seen in Figure 2(c), changing π can cause discrete jumps in the U2N duration when a worker changes the number of months spent searching for a non-college job. Changing ϕ impacts tightness and therefore the average unemployment duration, helping improve the model's fit to the U2N duration.

We now introduce the estimation procedure. Denoting $\tilde{m}$ (m) as the vector of 33 model generated (empirical) moments, the vector of 14 parameters, $\hat{\vartheta}$, is given by

$$\hat{\vartheta} = \arg\min \big(\tilde{m} - m\big)' W \big(\tilde{m} - m\big). \tag{11}$$

We use two weighting matrices, W. The first, and one we use throughout this section, is $W = I/m^2$, where I is the identity matrix. This scaled identity matrix minimizes the sum of squared percentage deviations between the model and empirical moments and does not place more weight on moments which are larger in magnitude. Table 3 and Figure 3 show that the model matches the data well. Table 4 displays the parameter values. We also use the inverse variance-covariance matrix of the empirical moments for W, which produces similar parameter values. See Appendix C.2 for more details.

[INSERT TABLE 3 AROUND HERE.]

[INSERT FIGURE 3 AROUND HERE.]

5.2. Decomposing Duration Dependence

This section evaluates the relative contributions of unobserved heterogeneity and human capital dynamics in generating duration dependence. Beginning with Figure 4(a), we present the transition path from the data, model, and model without skill dynamics during underemployment by setting $d_{n,\tau} = d_{c,\tau} = 0$. The model with only unobserved heterogeneity generates a substantial amount of duration dependence.

We next ask what percentage of the decline in the transition probability at each underemployment duration relative to the transition probability at $\tau = 1$ observed in the data can be explained the model with unobserved heterogeneity only. Figure 4(b) illustrates that the model with only unobserved heterogeneity explains at least 94% of the decline.

[INSERT TABLE 4 AROUND HERE.]

To arrive at an aggregate decomposition, we compute the weighted average of the fraction explained by unobserved heterogeneity over all τ. The weights are the fraction of underemployed workers who, in the steady-state, are employed at each τ. After removing human capital dynamics during underemployment, the model can explain 95.27% of the duration dependence. Therefore, unobserved heterogeneity is the primary driver of duration dependence in our model.[22]

[INSERT FIGURE 4 AROUND HERE.]

Next, we turn off the unobserved heterogeneity channel by setting $a^L = a^H = 1$ while all other parameters take the values in Table 4. Our main finding here is that workers never select into underemployment. Thus, both the duration dependence and existence of underemployment are closely tied to the presence of unobserved heterogeneity.

To conclude this section, we ask to what extent information frictions matter in generating underemployment and duration dependence. To answer this, we solve the version of our model where a worker's suitability type is public information (see Appendix E for details). Two findings emerge from removing information frictions. First, under the calibrated values in Table 4, broad-suitable workers never search for a non-college job. Therefore, the pool of underemployed workers contains only limited-suitability workers. Second, the full information model generates a negligible amount of duration dependence, as the transition probability decreases from 0.011 at $\tau = 1$ to 0.010 at $\tau = 24$. It follows that information frictions are a key ingredient in generating an amount of duration dependence commensurate with what is observed in the data.

[22] Appendix A.11 provides suggestive evidence for the presence of unobserved heterogeneity in the data.

5.3. Sorting and Bad Luck

To this point, our main quantitative finding is that a vast majority of the duration dependence observed in the data is accounted for by unobserved heterogeneity. A related and open question is what role bad luck plays in generating long underemployment durations. To evaluate the role of bad luck, we simulate the model and compare two groups of broad-suitable workers. The first group, "lucky", are those who find their first job within three months of entering the labor market. The second, "unlucky", are those who take more than three months to find their first job. Figure 5 compares the fraction of each month spent in each labor market status across the two groups. From Figure 5(a), unlucky workers gradually transition into underemployment after spending their first three months unemployed. Despite a slow start, the average underemployment duration among the unlucky (lucky) group is 5.58 (5.54) months, which suggests that there is little relation between the length of an unemployment and underemployment spell.

[INSERT FIGURE 5 AROUND HERE.]

Table 5 shows that, similar to the model, there is a weak correlation between unemployment and underemployment durations in the data.[23] This is consistent with the intuition above. Workers exit underemployment relatively quickly (slowly) if they are a broad-suitable (limited-suitable) worker, irrespective of their unemployment history.

[INSERT TABLE 5 AROUND HERE.]

To further understand the model generated correlation between a worker's unemployment and underemployment duration, recall that the identifying restriction pins down the skill growth and decay parameters. Moreover, the unobserved heterogeneity parameters are chosen to match the shape of the transition path and U2N duration. Through this identification strategy, we found $a^H = 0.354$ and $a^L = 0.023$, meaning that broad-suitable workers

[23] Appendix A.9 contains more details on the construction of Table 5.

exit underemployment at a much higher rate than limited-suitable workers. Moreover, the skill loss parameters are not large enough to generate lasting effects on the productivity of the worker. Hence, a long unemployment spell does not have persistent effects on an unlucky worker's chances to exit underemployment.

5.4. Discussion of Main Assumptions

To support the notion that college graduates are uncertain about their type and learn over time, we use the National Longitudinal Survey of Youth 1979 (NLSY79), where respondents indicated their expected occupation in 5 years and at age 35, allowing us to construct forecast errors between their actual and expected occupations. We report three main findings here and delegate the details to Appendix A.8. First, most college graduates make forecast errors regarding their future occupation. Second, those who make larger forecast errors are more likely to become underemployed, especially early in their career. The third finding is related to the correlation between graduates' actual and predicted forecast errors. Under the null hypothesis that workers know their type, the correlation between the actual and predicted errors is zero (Chahrour and Ulbricht, 2023). We find a statistically significant positive correlation, which not only suggests that graduates do not know their type, it indicates that they learn their type over time (Baley et al., 2022).

Our identifying restriction is that output is independent of a worker's suitability type. Under this assumption, the relationship between wages and a worker's employment history is driven by human capital dynamics only. Moreover, the identifying restriction acts to dampen the role of information frictions in generating duration dependence, as a worker's expected suitability does not impact the output of a suitable match.

Suppose instead that the production function in college jobs was given by $y_c^i(\tau)$ with $y_c^L(\tau) < y_c^H(\tau)$ for all $\tau \in T$. We show in Appendix A.10 that two crucial determinants of the duration dependence in college wages tied to the unobserved heterogeneity channel are (i) the rate at which beliefs evolve over the underemployment spell and (ii) the difference in output across suitability types. As shown in Proposition 3, if beliefs are low enough upon entering underemployment, then the beliefs follow a convex shape with a steep initial decline and then level off at higher values of τ. Appendix A.10 shows that we find little evidence in the data to support a convex pattern in the association between underemployment and wages in college jobs. On the second determinant, $y_c^H(\tau) - y_c^L(\tau)$, Figure A.7 shows that the decline in relative wages is minimal relative to that in the exit probability, suggesting that differences in output across suitability types are small. To further solidify this point, we calibrate a version of the model where $y_c^H(\tau) = \alpha y_c^L(\tau)$ and find $\alpha = 1.07$. Thus, the calibration assigns modest differences in productivity across suitability types. Moreover, we still find that unobserved heterogeneity explains the majority of duration dependence in underemployment (98.80%). See Appendix C.3 for the specifics on this calibration.

To summarize, we find that the magnitude and shape of duration dependence in wages for transitions from underemployment in the data do not support large differences in output across types or a strong correlation between wages and a worker's type. As such, relaxing the identifying restriction leads to a similar decomposition, where unobserved heterogeneity accounts for an even larger percentage of the duration dependence in underemployment. Finally, it should be noted that a decomposition of duration dependence in unemployment would be more sensitive to this identification strategy, as there is much more duration dependence in wages in transitions from unemployment than underemployment.

The ability to search on the job is an important avenue through which broad-suitable workers exit underemployment, and makes workers more willing to enter underemployment in light of the uncertainty they face about their suitability. Moreover, the search intensity of underemployed workers, λ, is influential in determining the amount of information underemployed workers acquire from not finding a college job.[24] The inclusion of on the job search is motivated by the fact that 79.5% of transitions from underemployment to proper

[24]Recall that underemployed workers update their beliefs according to $H(\lambda p, \mu)$ in (4). All else being equal, a higher λ implies a higher job finding probability of suitable workers, meaning workers will place more weight on the possibility they are a limited-suitable type if they do not find a college job.

employment in our NLSY97 sample occur through a job-to-job transition.

Finally, the model does not feature interactions between those with and without a college degree. While such interactions are important for the cyclicality of underemployment (Barnichon and Zylberberg, 2019), we abstract from including less-educated workers because the block-recursive nature of the equilibrium would result in workers self-selecting into different submarkets based on their educational attainment (and other observable characteristics). Therefore, the entry of firms and duration dependence in underemployment would not be impacted by having less-educated workers in the model.

One way to generate interactions between less- and highly educated workers is to depart from the directed search environments of Gonzalez and Shi (2010) and Menzio and Shi (2011). One could also assume output from non-college and college jobs are imperfect substitutes in the production of a final good. While these are interesting extensions, they would bring complications, as agents would need to keep track of the distribution of workers.

5.5. Robustness of Duration Dependence Decomposition

To assess the sensitivity of our main quantitative result, the decomposition of duration dependence, we conduct several robustness exercises.

Recall that we define a college job as an occupation where at least 50% of O*NET respondents indicate that a bachelor's degree or above is required to perform that job. We consider two alternative definitions. First, we adjust the threshold from 50% to 42.27% as this threshold corresponds to where 40% of occupations are classified as college occupations. With this definition in hand, we update the empirical moments, re-calibrate the model, and find that unobserved heterogeneity explains 92.96% of the duration dependence observed in the data. This is similar to our baseline result of 95.27%.

Next, we follow Barnichon and Zylberberg (2019) in using the 2012 Occupation Outlook Handbook published by the BLS to measure education requirements. The handbook lists the typical education needed for entry into each occupation. We define a college occupation as one in which a BA or above is typically required for entry. It is worth noting that 93.6% of occupational classifications (non-college vs. college) are the same as our baseline definition using O*NET. It is not surprising then that we find similar results. Namely, we find that unobserved heterogeneity explains 94.02% of the duration dependence. Appendix B shows the details behind the alternative definitions of non-college and college occupations, the empirical moments, calibrations, and full set of quantitative results under each definition.

The next robustness check uses estimates from Dinerstein et al. (2022) on the rate of skill depreciation and returns to experience instead of relying on the identifying restriction to calibrate the skill accumulation and loss parameters during underemployment. To summarize, Dinerstein et al. (2022) estimate an annual skill depreciation rate of 4.2% and net returns to working on productivity of 2.5%. After setting the skill loss and accumulation parameters, $d_{\chi,\tau}$ for $\chi \in X$, so that the model is consistent with the evidence from Dinerstein et al. (2022) and calibrating the rest of the parameters as in our baseline strategy, we find that unobserved heterogeneity explains 94.99% of duration dependence in underemployment. Appendix C.4.1 contains more particulars about this exercise.

Finally, recall that workers compensate firms for their recruiting costs through a one-time hiring fee. While our baseline analysis uses the wages under this contract to discipline the skill loss and accumulation parameters, we can also use the worker's effective wage. That is, the wage workers would earn if they instead paid a per-period fee to the firm. Suppose that the worker pays a fee $\tilde{\xi}_\chi(\upsilon, \tau)$ to the firm each period, where the present discounted value of the payments is equivalent to a one-time fee paid at the beginning of the match. The effective wage is $\tilde{w}_\chi(\upsilon, \tau) = y_\chi(\upsilon, \tau) - \tilde{\xi}_\chi(\upsilon, \tau)$. With the effective wages in hand, we re-calibrate the model and find that unobserved heterogeneity explains 98.36% of the duration dependence in underemployment. The specifics of the effective wages, calibration, and decomposition exercise can be found in Appendix C.4.2.

6. Conclusion

This paper has studied underemployment durations among recent college graduates in the US. Using the NLSY97, we have shown that the probability a worker exits underemployment decreases in their underemployment duration and that longer underemployment histories are associated with lower wages in college occupations.

To explain these facts, we developed a directed search model with unobserved heterogeneity, occupation-specific human capital, and on the job search. Workers learn about their job-finding probability in college jobs through search. Underemployment is generated when workers with a low expected suitability self-select into non-college jobs. Underemployed workers face both the accumulation of non-college and decay of college skills, creating structural duration dependence in underemployment.

A quantitative analysis shows that, through the lens of our model, unobserved heterogeneity is a large source of both the existence of underemployment and the duration dependence observed in the data. However, if the duration dependence in underemployment we have documented is driven by factors outside our model that are unrelated to wages (e.g., if underemployed workers become discouraged and no longer apply to college jobs), then true duration dependence could have a larger role than our findings suggest. We leave these interesting extensions to future research.

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Table 1: Frequency and Duration Across Labor Market Statuses

Labor force status	Unemployed	Underemployed	Properly employed
Ratio	0.031	0.392	0.522
Duration (months)	2.39	18.22	22.62

Notes: 5.6% of observations are outside the labor force.

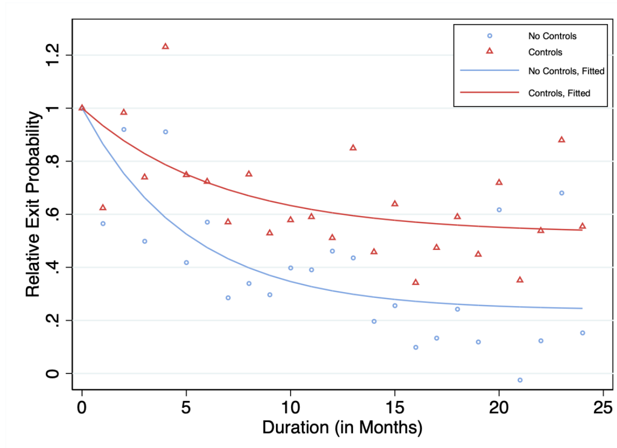
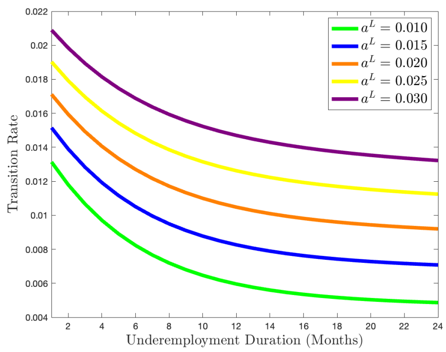


Figure 1: Duration Dependence in Underemployment

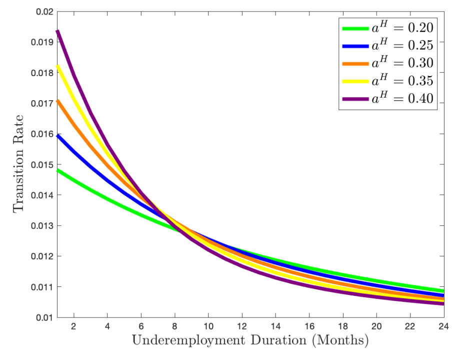
Table 2: Unemployment, Underemployment, and Wages

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0145***		-0.0145***	-0.0139***		-0.0136***
	(0.0009)		(0.0009)	(0.0011)		(0.0011)
Underhis		0.0003***	0.0003***		0.0007***	0.0006***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
Unhis × College				-0.0009		-0.0004
				(0.0013)		(0.0013)
Underhis × College					-0.0020***	-0.0019***
					(0.0002)	(0.0002)
Occupation (2-digit) FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.791	0.790	0.791	0.782	0.782	0.783

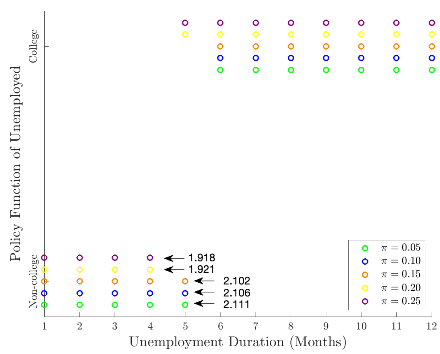
Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).



(a) Transition Path and a^L



(b) Transition Path and a^H



(c) U2N Duration and π

Figure 2: Identification of $\{\pi, a^L, a^H\}$

Table 3: Model and Data Comparison

Moment	Target	Model	Moment	Target	Model
Unemployment rate	0.081	0.081	$\partial \log(w_n)/\partial v$	-0.014	-0.014
Underemployment rate	0.416	0.414	$\partial \log(w_c)/\partial v$	-0.014	-0.014
U2N duration	2.147	2.111	$\partial \log(w_n)/\partial \tau$	0.001	0.001
College job premium	0.260	0.259	$\partial \log(w_c)/\partial \tau$	-0.001	-0.001
b/[Average labor productivity]	0.710	0.707	-	-	-

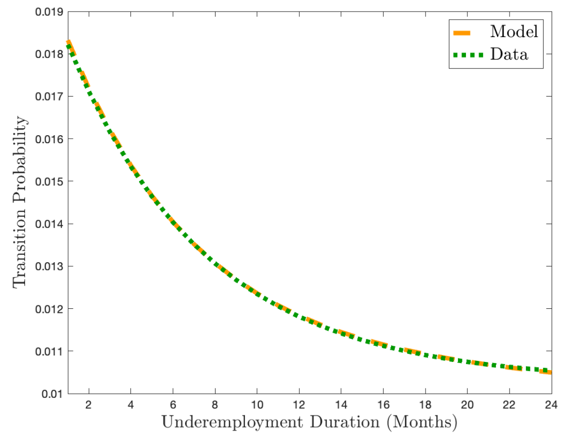
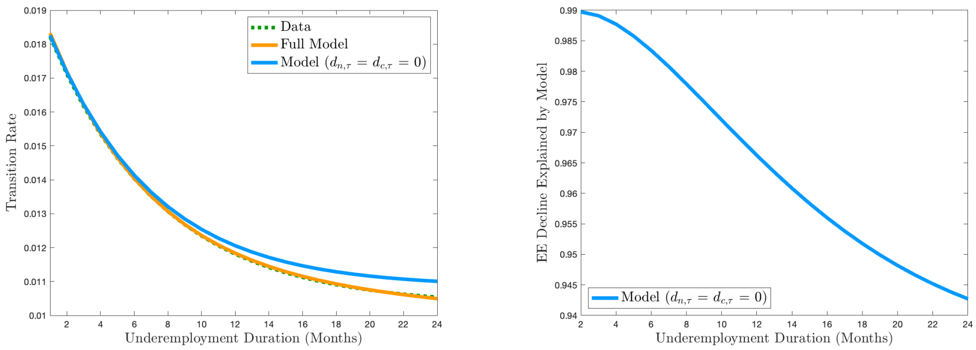


Figure 3: Duration Dependence in the Model and Data

Table 4: Parameter Values

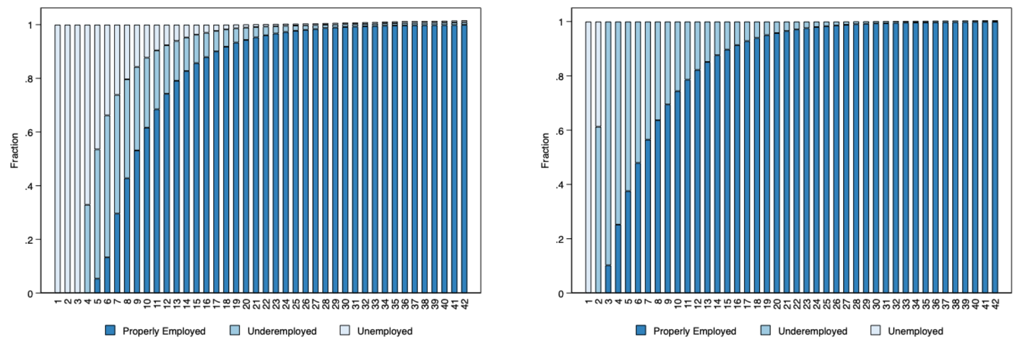
	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.354
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.049
g_n	Non-college productivity	0.745	ϕ	Pr. of regaining college skills	0.006
b	Utility while unemployed	0.611	$d_{c,\upsilon}$	College skill loss: unemp.	-0.014
k_n	Non-college vacancy cost	2.167	$d_{c,\tau}$	College skill loss: underemp.	-0.001
k_c	College vacancy cost	2.054	$d_{n,\upsilon}$	Non-college skill loss: unemp.	-0.014
λ	Employed search intensity	0.851	$d_{n,\tau}$	Growth of non-college skills	0.001



(a) Path of Transition Probabilities

(b) Fraction Explained at each τ

Figure 4: Duration Dependence Decomposition



(a) Unlucky Workers

(b) Lucky Workers

Figure 5: Percentage of Time Spent in Various Labor Market Statuses

Table 5: Correlation between Unemployment and Underemployment Durations

	Data		Model
	Unconditional	Conditional	
First U2N transition	-0.008 (0.854)	0.020 (0.898)	-0.025 (0.035)
All U2N transitions	-0.024 (0.446)	0.035 (0.742)	

Notes: p-values in parentheses. The model generated correlation and p-value is the average correlation and p-value across 100 simulations of our model, where each simulation simulates the labor market history of $10,000$ workers. The p-value within each simulation is obtained by testing the null hypothesis $\text{corr}(\upsilon, \tau) = 0$.