Supplementary Materials for "The Underemployment Trap"

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A Empirical Appendix

A.1 National Longitudinal Survey of Youth 1997 (NLSY97)

The NLSY97 tracks the lives of 8,984 individuals born between 1980 and 1984. It covers employment activities that can affect the ability to obtain and perform a job (such as education, training, etc.), as well as other sections on marriage, fertility, household composition, and health. The survey was conducted on an annual basis from 1997 through 2011 and biennially thereafter. All respondents were ages 12 to 17 at their first interview. As our analysis requires consecutive employment histories, we do not use any post-2011 employment records.

Our sample construction begins with 1,974 individuals who obtained a bachelor's degree or above and had at least one year of labor market experience before 2011. As the analysis of duration dependence requires consecutive employment records, 171 are dropped because of missing employment records in some weeks and 651 are dropped because they had a missing occupation code. Next, we drop 6 respondents who were always enrolled in school after obtaining a college degree. Finally, all employment records start from when the worker completely entered the labor market, leaving 996 respondents with at least one year of employment records after their last enrollment in school. Table A1 provides summary statistics.

The complete employment history includes weekly working hours, employment status, 2002 Census Industry and Occupation Codes, and real hourly wage.[1] The individual characteristics we focus on are gender, age, race, Armed Services Vocational Aptitude Battery (ASVAB) percentile, education history (including the highest education and the graduation date), college major, college GPA, student loan debt, and family income per-capita. As for the college major, we focus on primary majors self-reported term by term, and consider the major reported the most times during the undergraduate period as an individual's major. In terms of college GPA, we take the average GPA across all available reported terms. Finally, the financial information has been extracted to identify the amount of student loan currently owed and the family income per-capita, where the latter is calculated by dividing the total family income by the household size. We also consider the possibility of dual jobs and find that less than 13% of observations have more than one job. For these observations, we code the main job to be the one with the highest real wage.

[1] We deflate the hourly rate of pay with the Consumer Price Index. We then code the deflated hourly pay rate to missing if that is less than \$1 or more than \$1,000. Additionally, we code weekly working hours to missing if less than 10 or more than 98 hours. The average working hours per week among underemployed (properly employed) workers is 42.97 (45.25).

Table A1: Summary Statistics

	N	Mean	Std. Dev	Min	Max
Gender	996	0.56	0.50	0	1
– Male	439	0	-	-	-
– Female	557	1	-	-	-
Birth year	996	1982.06	1.40	1980	1984
– 1980	176	1980	-	-	-
– 1981	207	1981	-	-	-
– 1982	202	1982	-	-	-
– 1983	204	1983	-	-	-
– 1984	207	1984	-	-	-
Race	996	3.24	1.19	1	4
– Black	157	1	-	-	-
– Hispanic	135	2	-	-	-
– Mixed race (Non-Hispanic)	14	3	-	-	-
– Non-Black / Non-Hispanic	690	4	-	-	-
ASVAB percentile	882	69.25	23.11	3	100
Highest degree	1,112	4.42	0.73	4	7
– BA	740	4	-	-	-
– MA	214	5	-	-	-
– PhD	9	6	-	-	-
– Professional degree	33	7	-	-	-
Major	994	0.34	0.475	0	1
– Arts and Social Sciences	695	0	-	-	-
– STEM	299	1	-	-	-
Weekly hours	206,177	44.28	11.30	10	98
Real hourly wage ($)	206,872	17.61	21.59	1.01	519.65
Potential experience (months)	232,953	42.92	26.39	1	127
Student loan currently owed ($K)	232,953	0.26	2.60	0	120
Family income per-capita ($K)	212,420	37.71	38.62	0	421.37
College GPA	232,661	3.23	0.41	1.88	5
College occupation	214,029	0.569	0.495	0	1

A.2 Occupational Information Network (O*NET)

The O*NET measures occupational requirements and worker attributes. It is composed of four survey questionnaires covering skills, knowledge, generalized work activities, and work context. Respondents include job incumbents and occupational experts at various business work sites. Notably, O*NET reports the required level of education to perform a job under the domain of worker requirements, which enables us to determine whether an occupation typically requires a bachelor's degree or above.

A.3 Measurement of Occupational Skill Requirements

To measure the distance in skill requirements between occupations, we start by measuring the occupation's skill requirements along multiple dimensions. Specifically, each occupation is represented by a three-dimensional vector $(r_{verbal}, r_{math}, r_{social})$ where r_{verbal} measures the occupation's verbal skill requirement, r_{math} measures the math/quantitative skill requirement, and r_{social} captures the social skill requirement.

To measure verbal and mathematical skill requirements, we strictly follow the methodology used by Guvenen et al. (2020). The first step is to construct four scores for each occupation. The scores are: (i) word knowledge, (ii) paragraph comprehension, (iii) arithmetic reasoning, and (iv) mathematics knowledge. To construct these scores, we first select 26 O*NET descriptors that are chosen by the Defense Manpower Data Center (DMDC) and are listed at the top of Table A2. In the raw data, these descriptors range in value from 0 to 5. We re-scale their values in each year to fall between 0 and 1 and then take the average value for each descriptor between 2003 and 2011. Finally, we construct a weighted average in each of the four skill categories using the weighting matrix provided by the DMDC. For example, to construct the word knowledge score in occupation o, $S_{o,wk}$, we compute

$$S_{o,wk} = \sum_{i=1}^{26} s_{o,i} * \omega_{wk,i}, \tag{A.1}$$

Table A2: List of Descriptors

Panel A: Verbal and Math Skills		
Oral Comprehension	Written Comprehension	Deductive Reasoning
Inductive Reasoning	Information Ordering	Mathematical Reasoning
Number Facility	Reading Comprehension	Mathematics Skill
Science	Technology Design	Equipment Selection
Installation	Operation and Control	Equipment Maintenance
Troubleshooting	Repairing	Computers and Electronics
Engineering and Technology	Building and Construction	Mechanical
Mathematics Knowledge	Physics	Chemistry
Biology	English Language	
Panel B: Social Skills		
Social Perceptiveness	Coordination	Persuasion
Negotiation	Instructing	Service Orientation

where $s_{o,i}$ is descriptor i's average value between 2003 and 2011 for occupation o and $\omega_{wk,i}$ is the weight given to descriptor i in the category of word knowledge.

Second, we normalize the standard deviation of each score to one and reduce these four scores into two composite indicators, r_{verbal} and r_{math}, by applying principal component analysis (PCA). The verbal skill is the first principal component of word knowledge and paragraph comprehension, and the math skill is the first principal component of arithmetic reasoning and mathematics knowledge. The verbal and math skills are then converted into percentile ranks among all occupations.

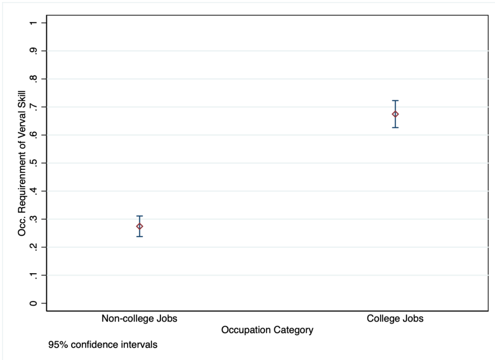
The social skill requirement can be identified similarly. By applying PCA to six scaled O*NET descriptors, we construct a single index to reflect the social skill requirement and then apply the percentile transformation as described above. The six descriptors used to construct the social skill requirement are listed in Panel B of Table A2. Based on the skill requirement along each dimension $(r_{verbal}, r_{math}, r_{social})$, we proceed to calculate the average skill requirement for each occupation by taking the unweighted average across the three dimensions.

Table A3: Skill Requirements of Five Most Common College/Non-college Jobs

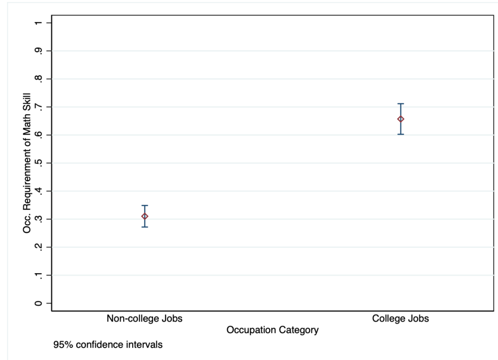
Occupation	Verbal	Math	Social	Avg.
Panel A: College jobs				
Elementary and middle school teachers	0.82	0.77	0.85	0.81
Registered nurses	0.76	0.67	0.80	0.74
Accountants and auditors	0.64	0.86	0.33	0.61
Secondary school teachers	0.84	0.81	0.92	0.85
Social workers	0.24	0.14	0.96	0.44
Panel B: Non-college jobs				
First-line supervisors/Managers of retail sales workers	0.33	0.44	0.56	0.44
Retail salespersons	0.10	0.21	0.20	0.17
Sales representatives, wholesale and manufacturing	0.28	0.37	0.67	0.44
Secretaries and administrative assistants	0.40	0.23	0.18	0.27
Customer service representatives	0.34	0.31	0.30	0.32

Next, we examine the relationship between skill and education requirements. Table A3 lists the mean skill and education requirements of the five most common college and non-college jobs in our sample. College jobs are typically associated with higher skill requirements along each skill dimension, as well as the average skill requirement. Figure A.1 further demonstrates this by plotting the average skill requirement among non-college and college jobs for verbal, math, social and average skill requirements.

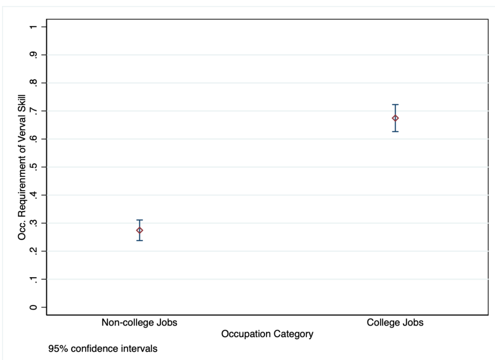
Finally, Figure A.2 presents a heat map demonstrating the correlation between skill and education requirements. Darker shades of red indicate a stronger positive correlation. The first column represents the percentage of respondents in the O*NET surveys who state that a bachelor's degree or higher is needed to perform a certain occupation. The second column is a binary variable that indicates whether at least 50% of respondents indicate that a bachelor's degree or higher is necessary to perform the occupation. Notably, it shows a positive correlation between education and skill requirements.



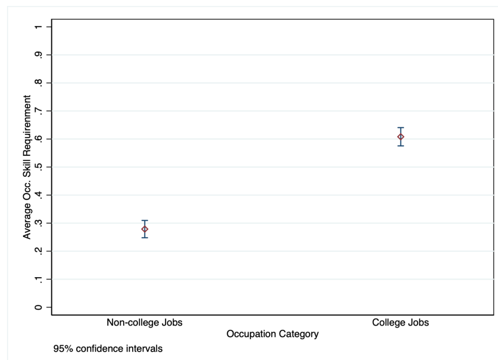
(a) Verbal



(b) Math



(c) Social



(d) Average

Figure A.1: Comparison of Skill Requirements

Notes: Graph shows 95% confidence intervals. We test the null hypothesis that the verbal/math/social/average skill requirement of non-college jobs is the same as that of college jobs against the alternative that the skill requirement of non-college jobs is below that of college jobs, and the test yields a p-value less than 0.01.

A.4 Within-firm Transitions

As discussed in the main text, there may be measurement error in within-firm occupational transitions. We attempt to correct this error by identifying "genuine" within-firm occupation switches from non-college to college occupations. To do so, we first measure the angular distance between the skill requirement of the current college occupation and the previous non-college occupation. Specifically, let $\phi : \mathbb{R}^3 \times \mathbb{R}^3 \to [0, \pi/2]$, and define the angular

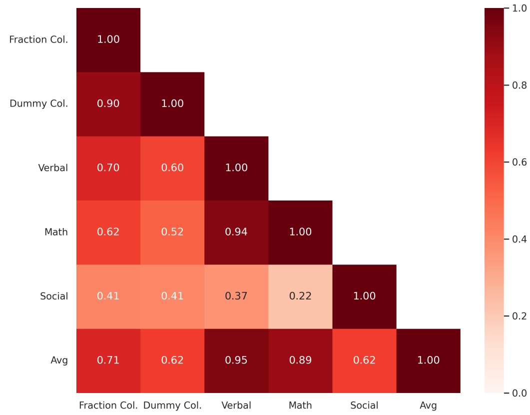


Figure A.2: Correlation Between Education and Skill Requirements

distance between two skill vectors $\mathbf{r}_i$ and $\mathbf{r}_j$ as:

$$\phi(\mathbf{r}_i, \mathbf{r}_j) = \cos^{-1}\left(\frac{\mathbf{r}_i \cdot \mathbf{r}_j'}{\|\mathbf{r}_i\|\|\mathbf{r}_j\|}\right). \tag{A.2}$$

A within-firm transition from an occupation i to another different occupation j is treated as a genuine transition if and only if $\phi(\mathbf{r}_i, \mathbf{r}_j) \geqslant \bar{\phi}$ where $\bar{\phi}$ is chosen so that the average correlation in skill requirements among genuine switches is close to zero. We set $\bar{\phi} = 18.094$, which results in a correlation in skill requirements among within-firm occupation switches of 0.0048. In our sample, 46/96, or 48% of within-firm transitions from non-college to college jobs are identified as genuine switches, which is close to the propensity of switching careers, 42.1%, obtained by Baley et al. (2022).

An alternative measure of distance between skill requirements is the Euclidean distance, which captures differences not only in the composition of skill requirements, but also in the magnitude of each skill requirement. The Euclidean distance between occupation i and j is given by:

$$\psi(\mathbf{r}_i, \mathbf{r}_j) = \sqrt{\sum_k (r_{i,k} - r_{j,k})^2}, \tag{A.3}$$

where $r_{i,k}$ is occupation i's requirement in aptitude $k \in \{verbal, math, social\}$.

A.5 Skill Distance, College Wages, and Underemployment History

To support the notion of the accumulation and decay of occupation-specific human capital, we study how the association between college wages and underemployment history varies with the distance in skill requirements between a worker's current college occupation and previous non-college occupation. The idea here is that if the distance in required skills between the two occupations is larger, then the skills required by the current college occupation would have been used less intensively in the previous non-college occupation and thus experienced a greater rate of decay, ultimately leading to a stronger association between underemployment history and wages in college occupations. To assess this hypothesis, we estimate the following regression:

$$w_{it} = \alpha \text{Underhis}_{it} + \gamma \phi_{it} + \zeta \text{Underhis}_{it} \times \phi_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \tag{A.4}$$

where ϕ_{it} is the distance in skill requirements between individual i's current college occupation and their most recent non-college occupation and X contains the same controls as in equation (3) in the main text. We use two measures of distance. The first is the Euclidean distance while the second is the angular distance as in Baley et al. (2022). The estimation of (A.4) only includes observations among individuals currently employed in a college occupation and who have been previously underemployed. Moreover, we restrict to those individuals where the average skill level in their current college occupation is higher than their previous non-college occupation.

Table A4 contains the results. Column (1) reveals that a larger Euclidean distance is associated with a significantly stronger relationship between the worker's underemployment history and wages in college occupations. Column (2) echoes this result when we use the angular distance.

Table A4: Skill Distance, College Wages, and Underemployment History

	(1)	(2)
Underhis	0.0173	0.0374***
	(0.0106)	(0.0098)
Underhis×Euclidean distance	-0.0454***	
	(0.0103)	
Underhis×Angular distance		-0.0017***
		(0.0004)
N	16,594	16,594
R^2	0.924	0.923

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

A.6 Duration Dependence with Different Sets of Control Variables

To elucidate the observable factors that attenuate the duration dependence profile, we take a closer look at the estimation in equation (2) by progressively incorporating control variables as follows:

$$
\begin{aligned}
y_{it} = {} & \beta\tau_{it} + \delta_t + \epsilon_{it} && \text{(Control 1)} \\
& + \text{Gender}_i + \text{Race}_i + \text{Edu}_i + \text{Gender}_i \times \text{Race}_i + \text{Gender}_i \times \text{Edu}_i + \text{Age}_{it} + \text{ASVAB}_i && \text{(Control 2)} \\
& + \text{Major}_i + \text{GPA}_i && \text{(Control 3)} \\
& + \text{FamInc}_{it} + \text{Loan}_{it} && \text{(Control 4)} \\
& + \text{JobSat}_{it}. && \text{(Control 5)}
\end{aligned}
$$

Figure A.3 presents the results. The red curve (Control Set 1) illustrates the transition path when controlling for year and month fixed effects. Control Set 2 additionally controls for gender, race, highest education, gender interacted with race, gender interacted with highest education, age bins and ASVAB scores, that produces the orange curve. We then add college major (Arts and Social Science versus STEM) and GPA bins, producing the yellow line under

Control Set 3. By further controlling for family income per-capita and student loan owed, the green curve represents the duration dependence under Control Set 4. Finally, the blue curve (Control Set 5) reveals a notable attenuation in duration dependence profile when we control for the current job satisfaction.

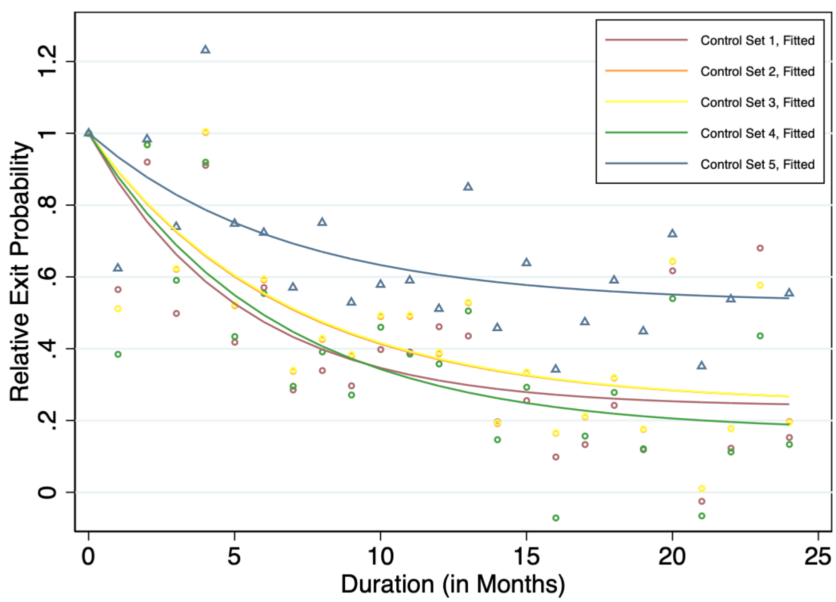


Figure A.3: Attenuation of the Duration Dependence Profile

A.7 Supplementary Tables and Figures

Table A5: Occupations around the 50% Threshold

Occupation Title	College Fraction
Geological and petroleum technicians	46.19
Other life, physical, and social science technicians	48.72
Sales representatives, wholesale and manufacturing	49.76
Designers	50.13
Directors, religious activities and education	50.97
Religious workers, all other	50.97
Cost estimators	51.03
Producers and directors	51.57
Construction managers	52.10
Judges, magistrates, and other judicial workers	53.40
Writers and authors	53.82
Other business operations specialists	54.15
Network systems and data communications analysts	54.49

Table A6: Top 10 College and Non-college Occupations

College Occupations	N	Non-college Occupations	N
Elementary/Middle school teachers	11,771	First-Line supervisors/Managers of retail sales workers	6,788
Registered nurses	5,990	Retail salespersons	3,806
Accountants and auditors	5,762	Sales representatives, wholesale and manufacturing	3,297
Secondary school teachers	5,761	Secretaries and administrative assistants	3,136
Social workers	4,703	Customer service representatives	2,978
Managers, all other	3,989	Police and sheriff's patrol officers	2,973
Financial managers	3,559	First-Line supervisors/Managers of office and administrative support workers	2,692
Other teachers and instructors	3,517	Waiters and waitresses	2,481
Computer software engineers	3,396	Cashiers	1,879
Marketing and sales managers	3,225	Loan counselors and officers	1,866

Table A7: Underemployment and College Major

Major	N	Respondents	Underemp. ratio
Panel A: Arts and Social Sciences			
Liberal arts and science	104	2	0.337
International relations and affairs	156	1	0.122
Social work	187	1	0.989
Archaeology	291	1	0.808
Hotel/Hospitality management	500	3	0.790
Pre-law	531	2	0.452
Human services, general	578	3	0.351
Home economics	595	4	0.606
Area studies	709	2	0.234
Anthropology	709	6	0.068
Theology/Religious studies	1,148	5	0.462
Philosophy	1,361	5	0.505
Foreign languages	2,244	8	0.311
English	4,786	24	0.335
Political science and government	5,422	26	0.375
Economics	5,636	16	0.265
History	5,832	32	0.482
Sociology	6,905	31	0.283
Criminology	7,022	31	0.573
Fine and applied arts	11,928	45	0.635
Psychology	16,015	69	0.398
Communications	17,910	68	0.442
Education	20,574	99	0.242
Business management	56,538	211	0.484
All Arts and Social Sciences	167,681	695	**0.429**
Panel B: STEM			
Pre-vet	156	1	0.865
Nutrition/Dietetics	365	2	0.399
Pre-med	448	4	0.213
Agriculture/Natural resources	2,163	8	0.626
Mathematics	2,621	13	0.491
Interdisciplinary studies	2,622	12	0.387
Physical sciences	3,131	16	0.262
Architecture/Environmental design	3,132	15	0.213
Nursing	6,378	28	0.035
Other health professions	7,982	38	0.393
Biological sciences	10,430	49	0.251
Computer/Information science	12,634	52	0.446
Engineering	13,096	61	0.312
All STEM	65,158	299	**0.325**

Notes: To compute the average underemployment ratio for Arts and Social Sciences and STEM, each major's ratio is weighted by its respective number of observations.

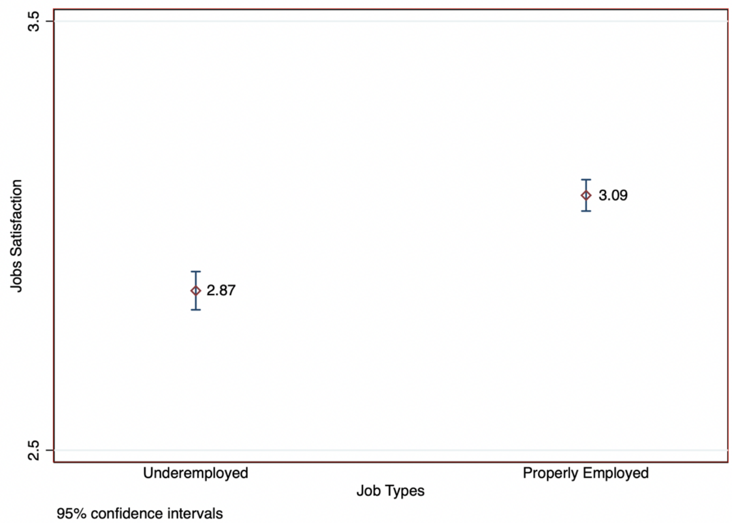
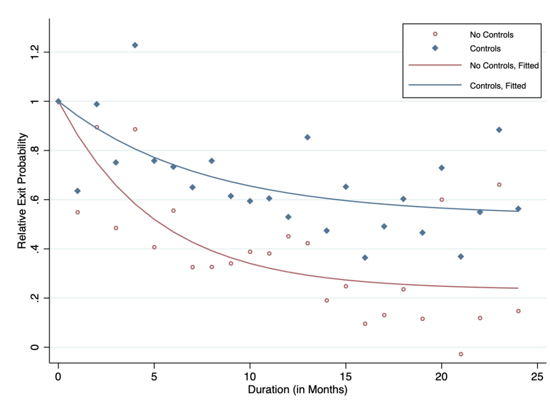
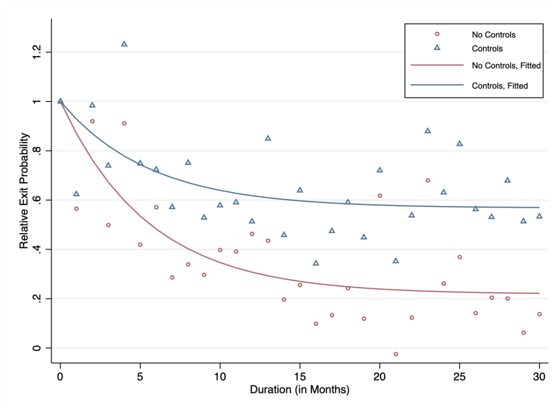


Figure A.4: Job Satisfaction among Underemployed and Properly Employed

Notes: We test the null hypothesis that the job satisfaction is equal among the underemployed and the properly employed against the alternative that the job satisfaction among the properly employed is different from the job satisfaction among the underemployed. The p-value is less than 0.01, indicating significant differences in job satisfaction between the two groups.



(a) Extra Transition Time



(b) Capped at 30 Months

Figure A.5: Additional Duration Dependence Profiles

Table A8: College Wages and Underemployment History (1-digit Industry and Occupation Codes)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0166***		-0.0164***	-0.0148***		-0.0145***
	(0.0010)		(0.0010)	(0.0011)		(0.0010)
Underhis		0.0006***	0.0006***		0.0010***	0.0009***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
Unhis × College				-0.0011		-0.0002
				(0.0013)		(0.0013)
Underhis × College					-0.0023***	-0.0022***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.778	0.777	0.778	0.774	0.774	0.774

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), regional and national annual unemployment rates, age, age squared, per-capita family income (\$$K$), student loan debt (\$$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 1-digit level), and regions.

Table A9: College Wages and Underemployment History (1-digit Occupation FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0154***		-0.0153***	-0.0153***		-0.0150***
	(0.0010)		(0.0010)	(0.0011)		(0.0011)
Underhis		0.0004***	0.0004***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
Unhis × College				0.0002		0.0008
				(0.0014)		(0.0014)
Underhis × College					-0.0022***	-0.0021***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172,149	172,149	172,149	172,149	172149	172149
R^2	0.784	0.784	0.784	0.784	0.784	0.785

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), regional and national annual unemployment rates, age, age squared, per-capita family income (\$$K$), student loan debt (\$$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 1-digit level), and regions.

Table A10: College Wages and Underemployment History (2-digit Occupation FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0145***		-0.0145***	-0.0144***		-0.0141***
	(0.0009)		(0.0009)	(0.0010)		(0.0010)
Underhis		0.0003***	0.0003***		0.0005***	0.0005***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
Unhis × College				-0.0004		-0.0000
				(0.0013)		(0.0013)
Underhis × College					-0.0017***	-0.0016***
					(0.0001)	(0.0001)
2-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.791	0.790	0.791	0.791	0.791	0.791

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), regional and national annual unemployment rates, age, age squared, per-capita family income ($\$K$), student loan debt ($\$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and regions.

Table A11: College Wages and Underemployment History (Year and Month FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0144***		-0.0143***	-0.0137***		-0.0134***
	(0.0009)		(0.0009)	(0.0011)		(0.0011)
Underhis		0.0003***	0.0003***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
Unhis × College				-0.0012		-0.0007
				(0.0013)		(0.0013)
Underhis × College					-0.0020***	-0.0019***
					(0.0002)	(0.0002)
2-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.792	0.791	0.792	0.783	0.783	0.784

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), annual regional unemployment rate, age, age squared, per-capita family income (\$$K$), student loan debt (\$$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for year, month, individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and regions.

A.8 Information Frictions and Underemployment

This appendix presents suggestive evidence that college graduates have uncertainty regarding their abilities, learn about their type over time, and that information frictions are a source of underemployment. To begin, we first construct a proxy measure of information frictions by leveraging a set of questions in the National Longitudinal Survey of Youth 1979 (NLSY79). Specifically, the NLSY79 asks the following questions to all respondents, one time, in their initial interview:

(i) What kind of work do you think you would be doing 5 years from now? If more than one occupation, what one kind of work do you think you would prefer?

(ii) What kind of work would you like to be doing when you are 35 years old?

Unfortunately, these questions were not asked in the NLSY97, our main dataset. To conduct this analysis on the NLSY79 cohort, we apply the same sample selection criteria that is applied to the NLSY97 sample, which leaves 444 college graduates with both a valid response to the 5 year horizon question and who were employed five years later, and 1,006 with a response to the occupational expectation at age 35 and identifiable realized occupation at age 35.

To construct a proxy measure of information frictions, we compare the skill requirements in a worker's realized occupation and their anticipated occupation, both at 5 years after the initial survey (short-term forecast) and when the respondent is 35 years old (long-term forecast). The occupational forecast error of individual i is given by the difference in required skills in aptitude $j \in \{v, m, s\}$ (verbal, math, social) between one's anticipated occupation ($\hat{s}_{i,j}$) and the realized occupation ($s_{i,j}$):

$$\overline{FCE}_i = \frac{\sum_{j \in \{v,m,s\}} |s_{i,j} - \hat{s}_{i,j}|}{3}. \tag{A.5}$$

Figure A.6 displays the distribution of short-term and long-term forecast errors.[2] Of the

[2] The violin plot illustrates the distribution characteristics of FCE, where the thick bar in the middle

444 college graduates, only 74 (or 17%) end up matched with their expected occupation in 5 years. Further, the average short-term forecast error is 0.22. Among the 1,006 graduates with a valid long-term forecast error, 146 (or 15%) are exactly matched to their expected occupation by age 35. Given the longer prediction horizon, the average long-term forecast error is 0.27.

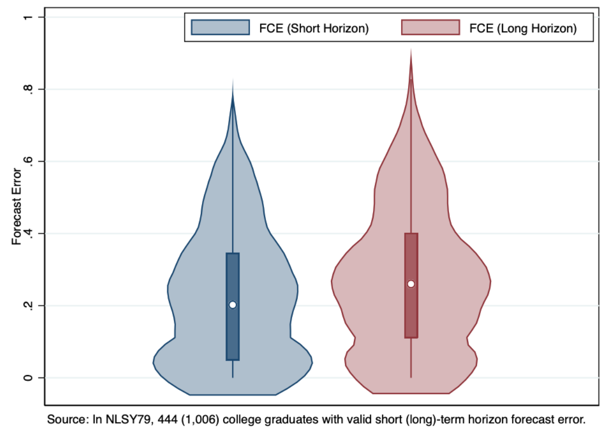


Figure A.6: Forecast Errors

In addition to these descriptive findings, we conduct a more formal test for the presence of information frictions, which follows Baley et al. (2022). Using the NLSY79, we define an individual i's forecast error in skill j between the realized occupation and predicted occupation at time $t + \Delta$ as:

$$FCE_{i,j,t} \equiv q_{i,j,t+\Delta} - \hat{q}_{i,j,t+\Delta}, \tag{A.6}$$

where t is the date of the initial interview, and Δ could be either 5 years or the duration from the time of the initial interview until the individual reaches the age of 35. Suppose that worker i knows their vector of skills across the j aptitudes, $\mathbf{a_i}$, and that skills are predictive of future occupations. With this in mind, one can predict the forecast error regarding the

represents the interquartile range of FCE. The thin lines extending from it signify the 95% confidence interval, and the white dot denotes the median of FCE.

utilization of skill j by computing:

$$\begin{aligned} PE_{i,j,t} &\equiv \mathbb{E}[FCE_{i,j,t}|\mathbf{a_i}], && \text{(A.7)} \\ &= \mathbb{E}[q_{i,j,t+\Delta}|\mathbf{a_i}] - \hat{q}_{i,j,t+\Delta}, && \text{(A.8)} \\ &= a_{i,j} - \hat{q}_{i,j,t+\Delta}. && \text{(A.9)} \end{aligned}$$

As $a_{i,j}$ and $\hat{q}_{i,j,t+\Delta}$ are both realized at the survey time t, the predicted error is realized at time t. Following Chahrour and Ulbricht (2023), the predicted and realized forecast errors are orthogonal to each other, $\text{Corr}(FCE_{i,j,t}, PE_{i,j,t}) = 0$, under the null hypothesis of full information. To examine whether the hypothesis of full information regarding workers' ability is supported by the data, we estimate the following regression:

$$\sum_{j \in v,m,s} FCE_{i,j,t} = \beta_0 + \beta_1 \sum_{j \in v,m,s} PE_{i,j,t} + \epsilon_{i,t}. \tag{A.10}$$

Additionally, we test the hypothesis along each skill aptitude j by estimating:

$$FCE_{i,j,t} = \beta_0 + \beta_1 PE_{i,j,t} + \epsilon_{i,j,t}. \tag{A.11}$$

Given that we have data on both short-term and long-term occupational expectations, we can test the hypothesis of full information over different horizons. Tables A12 and A13 present the results over a 5-year horizon or at age 35, respectively. In all cases, the coefficients β_1 are statistically significant at the 1% level, which leads us to reject the null hypothesis that workers have full information about their abilities.

Beyond the presence of information frictions, the statistically significant positive β_1 in Tables A12-A13 implies that workers learn their type over time. For example, if a worker underestimates her usage of verbal skill in the future, captured by a positive $PE_{i,t,v}$, our finding suggests that workers gain more certainty about their type, and tend to move towards a more verbal-intensive job than initially anticipated.

Table A12: Testing for Information Frictions (Expected Occupation in Five Years)

		Dependent Variable: $FCE_{i,j,t}$		
	(1)	(2)	(3)	(4)
	$\sum_j FCE_{i,j,t}$	Verbal	Math	Social
$\sum_j PE_{i,j,t}$	0.422***			
	(0.040)			
$PE_{i,j,t}$		0.326***	0.399***	0.387***
		(0.035)	(0.041)	(0.037)
N	444	444	444	444
R^2	0.219	0.168	0.201	0.199

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

Table A13: Testing for Information Frictions (Expected Occupation at Age 35)

		Dependent Variable: $FCE_{i,j,t}$		
	(1)	(2)	(3)	(4)
	$\sum_j FCE_{i,j,t}$	Verbal	Math	Social
$\sum_j PE_{i,j,t}$	0.540***			
	(0.027)			
$PE_{i,j,t}$		0.447***	0.530***	0.401***
		(0.024)	(0.027)	(0.024)
N	1,006	1,006	1,006	1,006
R^2	0.288	0.245	0.269	0.194

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

To further support information frictions as a source of underemployment, and given that the forecast error is a proxy for the workers' uncertainty regarding their type, we examine whether college graduates who made larger errors in forecasting their future occupation are more likely to end up underemployed upon their entry into the labor market. We estimate the correlation between the magnitude of forecast error and their probability of being underemployed in their first job by estimating:

$$Y_i^n = \beta_1 \overline{FCE}_i + \Gamma \cdot X_i + \epsilon_i. \tag{A.12}$$

The dependent variable, Y_i^n, is a dummy indicating if respondent i's first job is a non-college job or not. Alternatively, we use a dummy, Y_i^c, which indicates whether the first job is a college job as the dependent variable. The vector, X, contains gender, race, highest education, the interaction between gender and race, the interaction between gender and highest education, and the average skill level.

Table A14: Forecast Error and Underemployment

	Expectation in 5 Years		Expectation at Age 35	
	Y_i^n	Y_i^c	Y_i^n	Y_i^c
β_1	0.251**	-0.300**	0.148*	-0.184**
	(0.127)	(0.125)	(0.076)	(0.075)
N	444	444	1,006	1,006
R^2	0.044	0.063	0.087	0.097

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

Table A14 presents the results, from which two primary takeaways emerge. First, workers with a higher forecast error are more likely to become underemployed upon initially entering the labor market, supporting the argument that information frictions serve as a source of underemployment. Second, workers with more certainty typically transition directly to employment in college jobs. This aligns with our quantitative result that removing information frictions causes broad-suitable workers to only search for college jobs, resulting in lower underemployment.

Alternatively, we can explore the correlation between one's forecast error and the incidence of underemployment over the career by estimating:

$$Y_{i,t}^n(Y_{i,t}^c) = \beta_1\overline{FCE}_i + \beta_2 \text{Potexp}_{i,t} + \beta_3 \overline{FCE}_i \times \text{Potexp}_{i,t} + \Gamma \cdot X_i + \text{Month}_t + \text{Year}_t + \epsilon_{i,t}, \quad \text{(A.13)}$$

where $Y_{i,t}^n$ ($Y_{i,t}^c$) is a dummy indicating whether worker i is underemployed (properly employed) or not at time t and $\text{Potexp}_{i,t}$ is individual i's potential experience at time t. Equation (A.13) contains month and year fixed effects, in addition to the same individual level controls as in (A.12). In particular, β_1 captures the correlation between the forecast error and the probability of being underemployed (properly employed), while β_3 reflects how this correlation evolves over one's career.

Table A15: Forecast Error and Underemployment over the Career

	Expectation in 5 Years		Expectation at Age 35	
	$Y_{i,t}^n$	$Y_{i,t}^c$	$Y_{i,t}^n$	$Y_{i,t}^c$
β_1	0.4757***	-0.5525**	0.2769***	-0.3526***
	(0.0127)	(0.0127)	(0.0076)	(0.0078)
β_3	-0.0005***	0.0006***	-0.0002***	0.0003***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
N	130,152	130,152	347,099	347,099
R^2	0.0655	0.0742	0.0723	0.0812

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

Table A15 presents the results using the forecast error derived from the expected occupation in 5 years or at age 35. The primary finding is that workers who make a larger forecast error are more (less) likely to be underemployed (properly employed). Moreover, the impact of the forecast error diminishes over the course of one's career, as indicated by the opposite sign of β_3. This supports the notion of learning over one's career. As workers learn about their types over time, the initial uncertainty regarding their suitability becomes less influential.

A.9 Correlation between Unemployment and Underemployment Durations

This section details the estimation of the correlation between a worker's underemployment duration and preceding unemployment duration. Given that workers can transition from non-college to college jobs multiple times in data, we calculate the correlation both for all transitions and first transitions. To maintain consistency with the model, we assume (i) unemployment and non-employment are treated as equivalent, (ii) each college graduate enters the labor market with a one-month (or 4 weeks) period of unemployment, as workers in the model are unemployed for one period before they start searching for a job, and (iii) the unemployment (underemployment) duration is capped at 12 (24) months.

To compute the unconditional correlation, we first identify the unemployment duration prior to the underemployment spell. Then, we convert the duration (both prior unemployment and current underemployment duration) in weeks to months by taking each four weeks as one month, and cap the unemployment duration to 12 months and underemployment duration to 24 months. Subsequently, we compute the correlation between the underemployment duration and the previous unemployment duration.

For the conditional correlation, we start by converting weekly employment histories into a monthly basis by determining the primary labor force status for each month. Our criteria for this transformation are as follows: First, the labor force status most frequently reported within a month is regarded as the primary status for that period. Second, if the number of weeks in underemployment is identical to that in any other status (such as unemployment or proper employment), the month is classified as underemployed. Third, if the duration in weeks of unemployment equals that of proper employment, the month is categorized as unemployed. Based on the monthly employment data, we estimate:

$$\begin{aligned}\tau_{ik} = \beta_0 + \beta_1 \upsilon_{ik} &+ \text{Gender}_i + \text{Race}_i + \text{Edu}_i + \text{Gender}_i \times \text{Race}_i + \text{Gender}_i \times \text{Edu}_i \\ &+ \text{ASVAB}_i + \text{Major}_i + \text{GPA}_i + \text{AvgAge}_{ik} + \text{AvgFamInc}_{ik} + \text{AvgLoan}_{ik} \\ &+ \text{AvgSat}_{ik} + \text{StartYear}_{ik} + \text{StartMonth}_{ik} + \epsilon_{ik}.\end{aligned} \tag{A.14}$$

The dependent variable is the duration in months of the k^{th} underemployment spell for college graduate i, and υ_{ik} is the unemployment duration prior to the k^{th} underemployment spell. For the control variables, we consider gender, race, education, interactions of gender with race and education, ASVAB score in bins, college major, GPA in bins, average age, average family income, average outstanding student loan debt, average job satisfaction, the start year and month of the k^{th} underemployment spell.

A.10 More on Wages and Underemployment

This section takes a closer look into the relationship between underemployment and wages in college jobs. Under the employment contracts used throughout the paper and with output in college jobs given by $y_c^i(\tau)$, the average wage of workers in college jobs with history $\hat{\tau}$ relative to those with τ would be:

$$\frac{w_c(\hat{\tau})}{w_c(\tau)} = \frac{y_c^L(\hat{\tau}) + \mu_{\hat{\tau}}(y_c^H(\hat{\tau}) - y_c^L(\hat{\tau}))}{y_c^L(\tau) + \mu_{\tau}(y_c^H(\tau) - y_c^L(\tau))}. \tag{A.15}$$

From (A.15), two key determinants of the relative wages is the rate at which beliefs evolve, $\mu_{\hat{\tau}}/\mu_\tau$, and the difference in output across suitability types, $y_c^H(\tau) - y_c^L(\tau)$.

Next, we estimate the wage path of college jobs as a function of underemployment histories via the following regression:

$$w_{it} = \alpha + \sum_{\tau=1}^{24} \beta_\tau \times \mathbf{1}(\text{Underhis}_{it} = \tau) + \Gamma \cdot X_{it} + \delta_i + \epsilon_{it}, \tag{A.16}$$

where w_{it} is log wage in college jobs in time t. The right-hand side of (A.16) contains a series of dummy variables equal to 1 if the worker's underemployment history is equal to τ months for $\tau \in \{1, 2, \ldots, 24\}$, an individual fixed effect, and a vector of controls, X_{it}, that contains the same controls as in equation (3). Note that the college observations with $\tau = 0$ serves as the benchmark, so β_τ measures the effect of month τ of underemployment on wages in college jobs, relative to the effect of having zero months of underemployment history.

Figure 7(a) plots the estimated coefficients for β_τ (the blue circles). The dashed line shows the linear fit through the β_τ coefficients whereas the solid blue line represents the curve which is generated by estimating the following negative exponential model via weighted nonlinear least squares:

$$f(\tau) = a_1 + (1 - a_1)\exp(-b_1\tau). \tag{A.17}$$

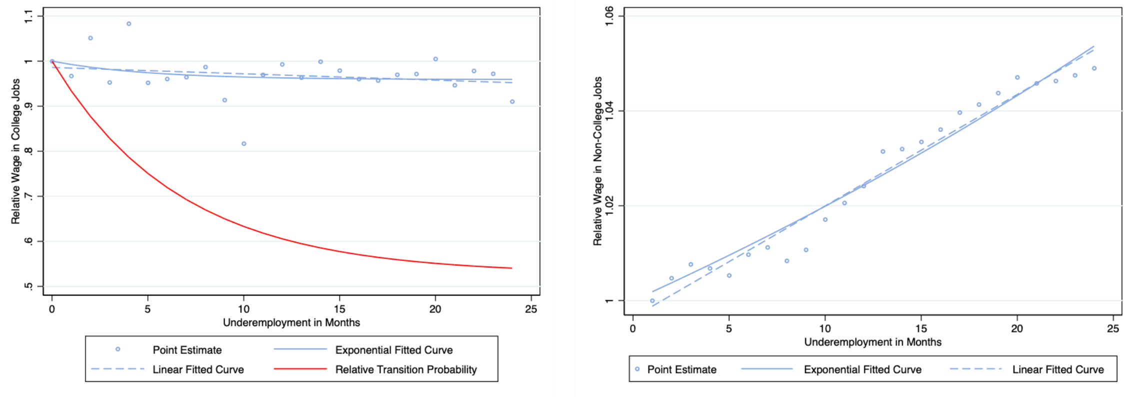
In (A.17), $f(\tau)$ is the relative wage at underemployment history τ to a worker with underemployment history $\tau = 0$, i.e. the coefficient of β_τ. Note that this model is the same as equation (1), and the solid red line in Figure A.7(a) reproduces the relative exit probabilities from underemployment to proper employment originally shown in Figure 1.

Figure A.7(b) presents the results from estimating the same regression specified by (A.16), except with wages in non-college jobs as the dependent variable. We also present the linear and exponential fitted patterns through the β_τ coefficients.

As a second exercise, we evaluate the effect of each month of underemployment history on wages in college jobs relative to non-college jobs by estimating:

$$\begin{aligned} w_{it} = \alpha + \sum_j \beta_j^n \times \mathbf{1}(\text{Unhis}_{it} = j) + \sum_k \beta_k^n \times \mathbf{1}(\text{Underhis}_{it} = k) + \sum_j \beta_j^n \times \mathbf{1}(\text{Unhis}_{it} = j) \times \text{College}_{it} \\ + \sum_k \beta_k^c \times \mathbf{1}(\text{Underhis}_{it} = k) \times \text{College}_{it} + \text{College}_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}. \end{aligned} \tag{A.18}$$

The dependent variable in (A.18) is individual i's log wage in time t, College is a dummy



(a) College Jobs (b) Non-College Jobs

Figure A.7: Wages and Underemployment History

for whether individual i is employed in a college occupation, δ_i is an individual fixed effect, and the vector X contains the same controls as in equation (3) and (A.16). Equation (A.18) also includes dummies for each unemployment and underemployment history.

We are particularly interested in the coefficients β_k^c, as these capture the effect of the k^{th} month of underemployment on wages in college jobs relative to non-college jobs. Figure A.8 presents the results from estimating (A.18) with $j \in \{0, 1, 2, \ldots 9\}$ and $k \in \{1, 2, \ldots, 60\}$. The scatter points represent the estimated coefficients, β_k^c, while the lines represent several fits through the scatter points. The blue line is the linear fit, the light-brown line is a quadratic fit, and the green line is a cubic fit. Finally, the red line is the result of estimating a locally weighted regression of β_k^c on the underemployment history, k.

A.11 Suggestive Evidence on Unobserved Heterogeneity

The results in Section 5 suggest that unobserved heterogeneity in workers' suitability for college jobs plays a large role in generating duration dependence in underemployment. In this section, we provide two sources of suggestive evidence from hourly wage data in the NLSY to support the presence of unobserved heterogeneity among college graduates. While wages are not a function of a suitability in the baseline theory, our model has several natural

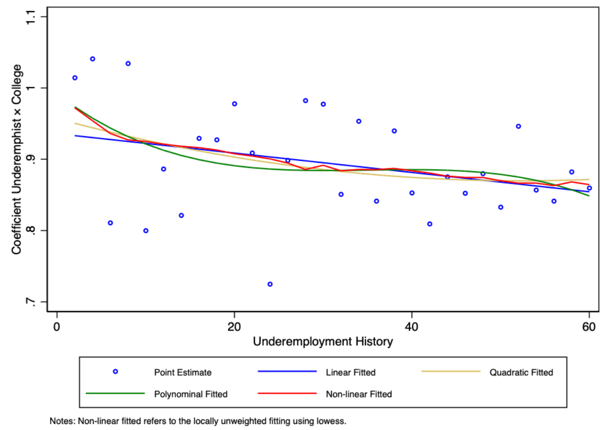


Figure A.8: Linear and Non-Linear Fits through Estimates of β_k^c Coefficients

implications for comparing wages in college occupations between workers who experience short- and long-underemployment spells.

First, suppose that wages were a function of a worker's suitability type. This would occur, for example, if limited suitability workers produced less output than broad-suitable workers. If this were the case, then the amount of residual wage dispersion among workers who transition from non-college to college jobs at relatively short underemployment durations would be larger than the amount of dispersion among the group who take longer to transition out of underemployment. In other words, there should be more wage inequality among observationally equivalent workers in the former group than the latter because the latter is primarily comprised of limited suitability workers. To investigate this, we estimate residual wage inequality in jobs held after a worker exits underemployment. The approach, which follows Acemoglu (2002), starts by estimating:

$$w_{it} = \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \tag{A.19}$$

where w_{it} is individual i's log hourly real wage at time t and X is a set of controls that includes a cubic in years of potential experience, the annual national unemployment rate, the annual regional unemployment rate, a quadratic in age, family income per-capita, out-

Table A16: Residual Wage Dispersion

	90/50 ratio	50/10 ratio
Full sample	1.525	1.568
Panel A: After the first underemployment spell		
<1 year to exit underemployment	1.324	1.306
$\geqslant 1$ year to exit underemployment	1.201	1.196
Panel B: Proper employment spell following underemployment		
<1 year to exit underemployment	1.319	1.315
$\geqslant 1$ year to exit underemployment	1.237	1.238

standing student loan debt, current level of job satisfaction, region, and 2-digit occupation and industry fixed effects. Finally, δ_i is an individual fixed effect. After estimating (A.19), we compute the ratio of the 90th to 50th and 50th to 10th percentile of the residuals.

The first row of Table A16 shows the 90/50 and 50/10 ratios for our entire sample. We see that the 90/50 ratio is 1.53 and the 50/10 ratio is 1.57, which is slightly below the typical range of 1.7-1.9 (Hornstein et al., 2011). Panel A restricts the sample to wages earned in college jobs following a worker's first underemployment spell and shows that the 90/50 (50/10) ratio is 12% (11%) larger in the group that exits underemployment in less than one year. Panel B shows that this pattern also emerges when we focus on the proper employment spell which immediately follows a spell of underemployment.

Our second form of evidence compares wage growth in college jobs between workers who experience short and long underemployment spells. The intuition is the following. If broad-suitable workers have a higher ability to learn new skills, then wage growth in college jobs should be higher among the group of workers who experience short underemployment durations, as this group contains most of the broad-suitable workers who experience underemployment. To investigate this, we estimate the following regression:

$$\Delta w_{it} = \beta \text{Long}_{it} + \Gamma \cdot X_{it} + \varepsilon_{it}, \tag{A.20}$$

Table A17: Wage Growth After Exiting Underemployment

	(1)	(2)	(3)	(4)
Long	-0.0233**	-0.0230**	-0.0224**	-0.0233**
	(0.0091)	(0.0091)	(0.0096)	(0.0099)
N	1,815	1,815	1,779	1,779
R^2	0.025	0.029	0.029	0.030

Notes: Standard errors are clustered at the individual level. The first specification consider a cubic in potential experience, highest level of education, race, gender, age, age square, as well as yearly, 2-digit occupational and industrial fixed effects. The second specification additionally controls for the one-year lagged unemployment rate. The third specification additionally controls for the marital status. The last specification additionally consider the interaction between the marital status and age, and its interaction with gender. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

where Δw_{it} is the difference in the log of average hourly real wage between quarter $t-1$ and quarter t in the worker's first proper employment spell and Long is equal to 1 if the worker's first underemployment spell lasted a year or longer and 0 otherwise. The vector X includes a cubic in potential experience, highest level of education, race, gender, age, age squared, and 2-digit occupation and industry fixed effects. Table A17 contains the results and shows, across all specifications, workers who experience longer underemployment spells exhibit a lower wage growth in college jobs than their observationally equivalent peers.

B Sensitivity Tests: Alternative Definitions of College Jobs

In the baseline analysis, college jobs are defined as those occupations where at least 50% of respondents indicate that a bachelor's degree or above is required to perform the job. For brevity, we label this as the "ONET50.00" definition. To ensure that our primary findings are not sensitive to a particular threshold, we explore alternative criteria for identifying college jobs.

B.1 Alternative Definitions

Our first alternative uses the O*NET descriptors but adopts a lower threshold. Specifically, we redefine college jobs as those occupations in which at least 42.27% of respondents indicate a bachelor's degree or above is necessary. This definition, henceforth referred to as the "ONET42.27", corresponds to the 60^{th} percentile in the empirical cumulative density function of college fraction across 298 distinct occupations, as depicted in Figure B.1. Under the ONET42.27 definition, 120 occupations are identified as college jobs, compared to 108 college occupations under the ONET50.00 criteria. Such an expansion could lead to more transitions from underemployment to proper employment, potentially attenuating the observed magnitude of duration dependence.

The second approach, which follows Barnichon and Zylberberg (2019), employs the variable *typical education needed for entry* reported by the 2012 Occupation Outlook Handbook, issued by the U.S. Bureau of Labor Statistics, henceforth labelled as the "OOH2012" definition. In particular, the 2012 Occupation Outlook Handbook details the typical entry education for 820 distinct SOC2010 occupations, based on federal and state regulations as well as the typical path of entry into a job.[3] Under this definition, an occupation is con-

[3] The OOH2012 lists eight entry education levels: (i) Less than high school; (ii) Postsecondary non-degree award; (iii) High school diploma or equivalent; (iv) Some college, no degree; (v) Associate's degree; (vi) Bachelor's degree; (vii) Master's degree; and (viii) Doctoral or professional degree.

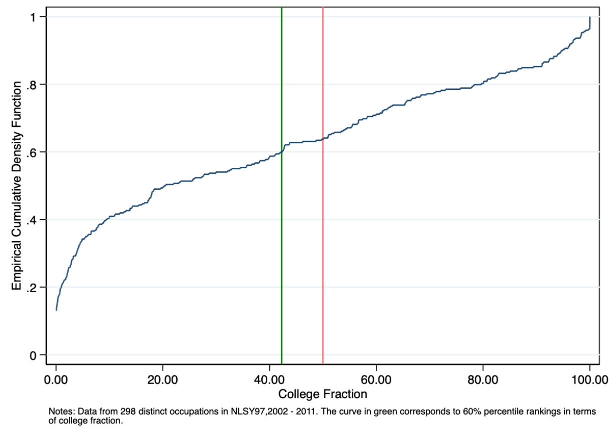


Figure B.1: Empirical CDF of College Fraction

sidered to be a college job if its typical education needed for entry is a bachelor's degree or above.

To identify the typical entry education for each SOC2002 occupation in our sample, we convert the occupation codes from the SOC2010 (used in OOH2012) to the SOC2002 (used in NLSY97). Next, we identify the entry education for each observed SOC2002 code. If a single SOC2002 occupation code corresponds to multiple SOC2010 codes, we compute the average education level for those SOC2010 occupations, and then take the education level that is closest to the computed average education level as its typical entry education.[4] Last, we manually adjust the education requirements for four SOC2002 occupations as most of their corresponding SOC2010 occupations require at least a bachelor's degree, even if their average entry education is below a bachelor's degree.[5] After determining the typical entry education for each SOC2002 occupation in our NLSY97 sample, we construct a binary variable to indicate whether an occupation is a college job or not.

Compared with the ONET50.00 identification, among the 298 occupations in the sample,

[4]Take the SOC2002 job "Claims Adjusters, Appraisers, Examiners, and Investigators" as an example. It corresponds to two distinct SOC2010 occupations: Insurance Appraisers, Auto Damage and Claims Adjusters, Examiners, and Investigators. The entry educations are post-secondary non-degree award and high school diploma or equivalent separately. The average education level would be 2.5, so its entry education level would be high school diploma or equivalent.

[5]These four SOC2002 occupations are (i) Other Teachers and Instructors, (ii) Designers, (iii) Miscellaneous Community and Social Service Specialists, and (iv) Emergency Management Specialists.

279 (93.6%) are consistently identified using the OOH2012 definition. To be specific, 99 out of 108 (91.67%) ONET50.00 college occupations are also considered as college occupations by OOH2012 definition. Additionally, 180 out of 190 (94.74%) ONET50.00 non-college occupations remain classified as non-college occupations under the OOH2012 definition. Meanwhile, 197,888 out of 214,029 (92.5%) of employment observations are unaffected when adopting this alternative definition.[6]

B.2 Empirical Evidence

In this section, we compare the empirical evidence in the main text with each alternative definition of college jobs. Overall, these comparisons show that the nature of underemployment, especially the negative duration dependence, is not sensitive to the definition of a college job.

B.2.1 The Prevalence and Persistence of Underemployment

To examine whether the underemployment remains prevalent and persistent across different definitions, we replicate the exercise in the main text and look into the fraction of respondent's history and duration length spent in each labor force status. Table B.1 shows that the average underemployment ratio and underemployment durations are similar across the three definitions.

B.2.2 Duration Dependence

To investigate how the magnitude of duration dependence reacts to different definitions, we re-estimate the duration dependence for each alternative definition using equation (2) and

[6] Here are some more details on the misalignment between the ONET50.00 and OOH2012 definitions in identifying college jobs where 19 (or 6.4%) occupations have different classifications. In particular, the OOH2012 classifies 10 of these occupations as college jobs, in contrast to their non-college job classification under the ONET50.00, a discrepancy we refer to as a Type 1 Misalignment. On the other hand, 9 occupations are identified as non-college jobs by the OOH2012 but as college jobs by the ONET50.00, a Type 2 Misalignment. Regarding the impact on employment observations in our sample, approximately 7.5% (16,141/214,029) are affected by these differences. This includes $6,014$ observations (2.8%) affected by a Type 1 Misalignment and 10,127 observations (4.7%) impacted by a Type 2 Misalignment.

Table B.1: Comparisons – Labor Force Statuses

	ONET50.00	ONET42.27	OOH2012
Ratio			
Unemployed	0.031	0.031	0.031
Underemployed	0.392	0.367	0.408
Properly Employed	0.522	0.547	0.505
Duration (months)			
Unemployed	2.39	2.39	2.39
Underemployed	18.22	17.62	18.70
Properly Employed	22.62	22.71	22.36

compare these estimates to the duration dependence identified under ONET50.00. Figure B.2 shows that the magnitude of duration dependence is similar in each definition.

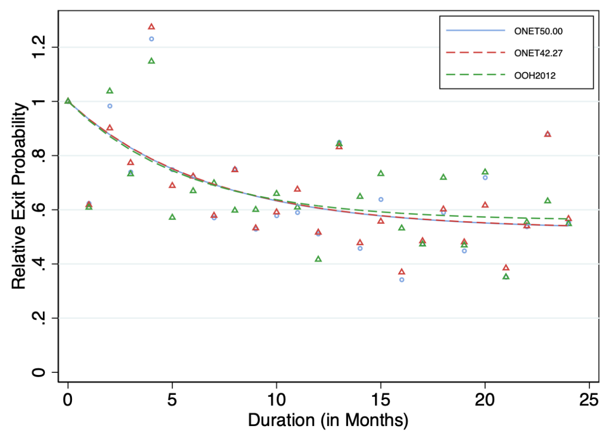


Figure B.2: Comparisons – Duration Dependence in Underemployment

B.2.3 Wages and Underemployment

To compare wage losses in college jobs across different definitions, we re-estimated the coefficients in equation (3) and present them in Table B.2. Specifically, we find that wage losses in college jobs become more severe under the alternative definitions. Given that the magnitude of wage losses is informative of skill evolution parameters, a larger wage loss indicates a greater depreciation of college skills during underemployment, which potentially enhances the role of human capital dynamics in determining the duration dependence in underemployment. Consequently, we proceed to re-calibrate the model by targeting these new data moments.

Table B.2: Comparisons – Wages and Underemployment

	ONET50.00	ONET42.27	OOH2012
$\partial \log(w_n)/\partial \upsilon$	-0.0136	-0.0118	-0.0135
$\partial \log(w_c)/\partial \upsilon$	-0.0136	-0.0182	-0.0162
$\partial \log(w_n)/\partial \tau$	0.0006	0.0006	0.0006
$\partial \log(w_c)/\partial \tau$	-0.0013	-0.0019	-0.0016

B.3 Quantitative Analysis

In this section, we replicate the quantitative analysis for the alternative definitions of college jobs.

B.3.1 Calibration

We start with re-calibrating of the model by targeting the data moments computed under each alternative definition. These calibrations follow the same strategy as stated in Section 5. Regardless of the definition used, the model can match the targeted moments well. The comparisons of the transition path observed in the data with the model-generated transition path are depicted in Figures B.3(a) and B.4(a) while Table B.3 details the model fits for other targeted moments. Last, the calibrated parameters by each definition are listed in Table B.4.

B.3.2 Decomposing Duration Dependence

With the calibrated models in hand, we can move on to explore the contribution of unobserved heterogeneity versus human capital dynamics to duration dependence in underemployment. To do so, we compute the duration dependence after turning off the human

Table B.3: Model Fits - Other Moments

	ONET50.00		ONET42.27		OOH2012	
	Target	Model	Target	Model	Target	Model
Unemployment rate	0.081	0.081	0.081	0.081	0.081	0.081
Underemployment rate	0.416	0.414	0.388	0.388	0.433	0.432
b/[Average labor productivity]	0.710	0.707	0.710	0.709	0.710	0.709
U2N duration	2.147	2.111	2.147	2.126	2.111	2.083
College job premium	0.260	0.259	0.243	0.246	0.275	0.280
$\partial \log(w_n)/\partial \upsilon$	-0.014	-0.014	-0.012	-0.012	-0.014	-0.014
$\partial \log(w_c)/\partial \upsilon$	-0.014	-0.014	-0.018	-0.018	-0.016	-0.016
$\partial \log(w_n)/\partial \tau$	0.001	0.001	0.001	0.001	0.001	0.001
$\partial \log(w_c)/\partial \tau$	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002

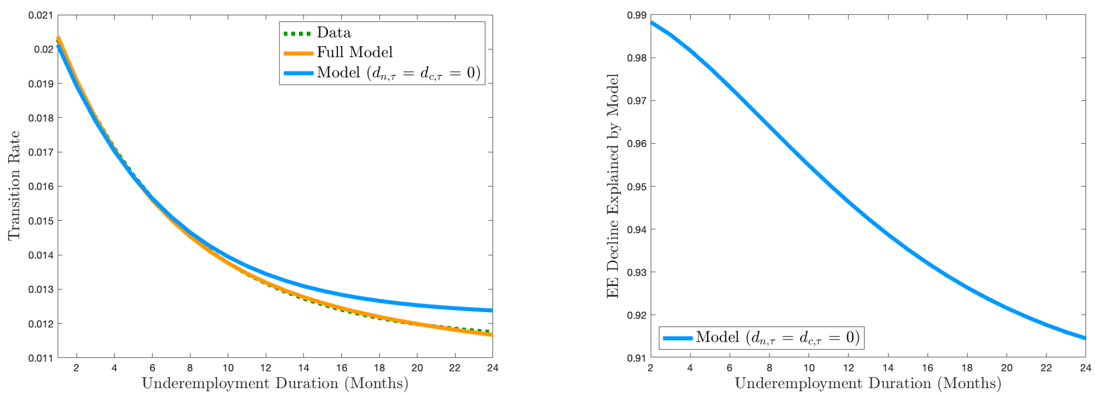
Table B.4: Calibrated Parameters

Parameter	Definition	ONET50.00	ONET42.27	OOH2012
β	Discount factor	0.996	0.996	0.996
δ	Entry/exit probability	0.011	0.011	0.012
g_c	College productivity	1.000	1.000	1.000
g_n	Non-college productivity	0.745	0.743	0.725
b	Utility while unemployed	0.611	0.612	0.601
k_n	Non-college vacancy cost	2.167	2.414	1.954
k_c	College vacancy cost	2.054	1.831	2.069
λ	Employed search intensity	0.851	0.856	0.852
a^L	Suitability pr.: type L	0.023	0.025	0.024
a^H	Suitability pr.: type H	0.354	0.354	0.405
π	Pr. of being a type H worker	0.049	0.051	0.037
ϕ	Pr. of regaining college skills	0.006	0.006	0.006
$d_{c,\upsilon}$	College skill loss: unemp.	-0.014	-0.019	-0.017
$d_{c,\tau}$	College skill loss: underemp.	-0.001	-0.002	-0.002
$d_{n,\upsilon}$	Non-college skill loss: unemp.	-0.014	-0.012	-0.014
$d_{n,\tau}$	Growth of non-college skills	0.001	0.001	0.001

capital dynamics channel. Figure B.3 displays the decomposition for the ONET42.27 definition. Notably, the model with only unobserved heterogeneity accounts for at least 91.5%

of this decline.

To arrive at the aggregate decomposition, we calculate the weighted average of the fraction explained by unobserved heterogeneity across all durations τ. The weights are determined by the proportion of underemployed workers in steady-state at each duration τ. The model without human capital dynamics explains 93.0% of the duration dependence. Similarly, Figure B.4 presents the decomposition results for the OOH2012 definition. In this case, the model with only unobserved heterogeneity explains at least 92.5% of the decline. On the aggregate, the model without human capital dynamics accounts for 94.0% of the duration dependence.



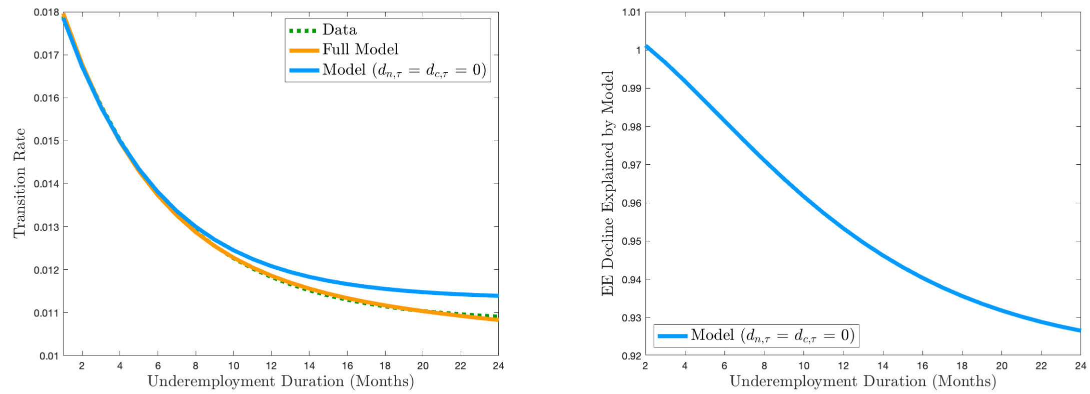
(a) Transition Probability Path

(b) Fraction Explained at each τ

Figure B.3: ONET42.27 Duration Dependence Decomposition

Additionally, we examine the extent to which information frictions contribute to the generation of underemployment and its duration dependence by analyzing the model with full information that still allows for heterogeneity in worker suitability and shocks to occupation-specific human capital. Figures B.5-B.6 illustrate the duration dependence with and without information frictions for the alternative definitions. The results in both definitions mirror the patterns observed in the baseline exercise.

First, in the model of full information where workers' types are publicly known, workers with broad suitability do not search for non-college jobs. Second, a mild duration dependence is still noticeable among workers with limited suitability who seek non-college jobs and be-

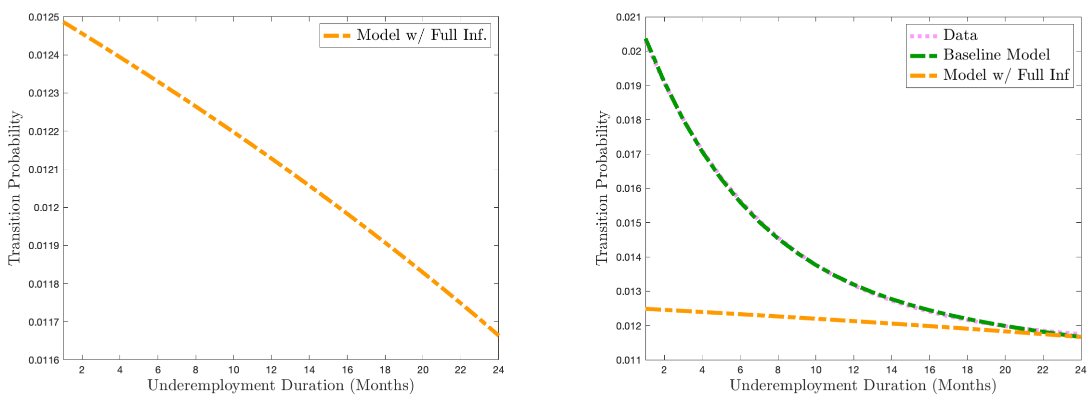


(a) Transition Probability Path

(b) Fraction Explained at each τ

Figure B.4: OOH2012 Duration Dependence Decomposition

come stuck in them. However, the magnitude of duration dependence becomes much smaller compared to what is observed in both the full model and the data. These observations collectively indicate that the information friction is crucial in generating the underemployment and the observed duration dependence in underemployment.



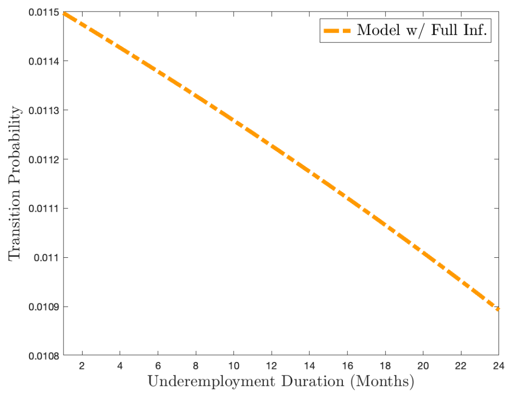
(a) No Information Frictions

(b) Model and Data Comparisons

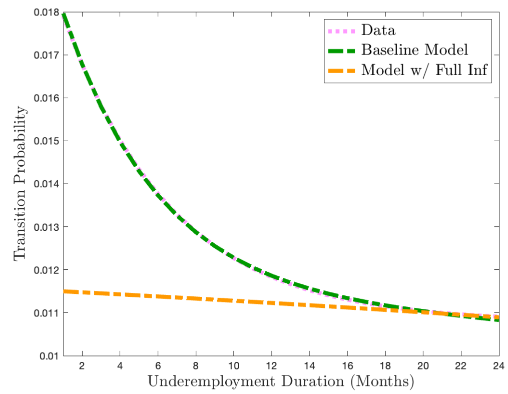
Figure B.5: ONET42.27 Duration Dependence with and without Information Frictions

B.3.3 Sorting and Bad Luck

So far, our comparisons indicate that unobserved heterogeneity among college graduates accounts for the majority of the duration dependence observed in underemployment. This

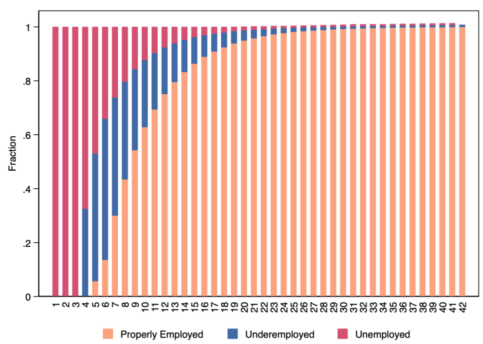


(a) No Information Frictions

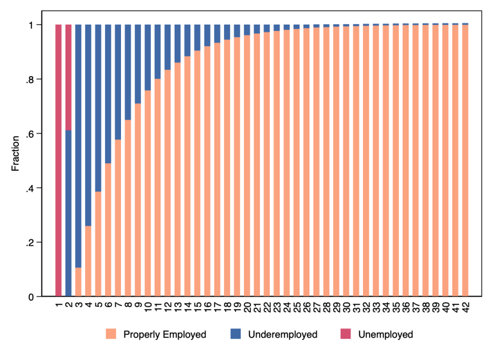


(b) Model and Data Comparisons

Figure B.6: OOH2012 Duration Dependence with and without Information Frictions



(a) Unlucky Workers



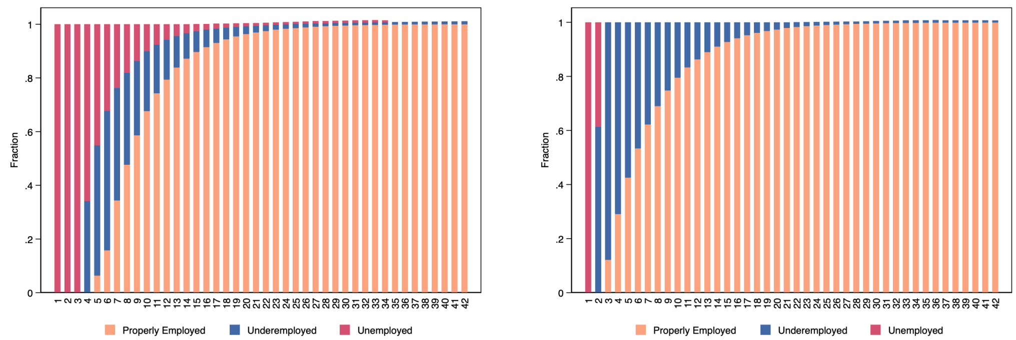
(b) Lucky Workers

Figure B.7: ONET42.27 Percentage of Time Spent in Various Labor Market Statuses

section replicates the exercise presented in the Section 5.3, which attempts to disentangle between sorting and bad luck in generating long underemployment spells.

Figure B.7 compares the fraction of each month spent in each labor market status between lucky and unlucky workers under the ONET42.27 definition. The average duration of underemployment for the unlucky (lucky) group is 5.41 (5.36) months. Similarly, Figure B.8 demonstrates the time allocation in each labor market status for the two groups under the OOH2012 definition, where the average underemployment duration for the unlucky (lucky) group is 4.88 (4.86) months.

Table B.5 presents the correlation between the duration of underemployment and its



(a) Unlucky Workers (b) Lucky Workers

Figure B.8: OOH2012 Percentage of Time Spent in Various Labor Market Statuses

preceding unemployment duration. Notably, under each definition, the correlation between the two durations is very close to zero, just as we observe in the data.

Table B.5: Correlation between υ and τ

	Data		Model
	Unconditional	Conditional	
ONET42.27 Definition			
First U2N transition	-0.005 (0.903)	0.016 (0.919)	-0.021 (0.093)
All U2N transitions	-0.026 (0.424)	0.046 (0.678)	
OOH2012 Definition			
First U2N transition	-0.027 (0.515)	0.060 (0.725)	-0.024 (0.055)
All U2N transitions	-0.037 (0.234)	0.034 (0.747)	

Notes: p-values in parentheses. The model generated correlation and p-value is the average correlation and p-value across 100 simulations of our model, where each simulation simulates the labor market history of $10,000$ workers. The p-value within each simulation is obtained by testing the null hypothesis $\text{corr}(\upsilon, \tau) = 0$.

C Calibration Appendix

C.1 Data Moments for Calibration

C.1.1 Wage Premium

To obtain this target, we estimate the following regression:

$$w_{it} = \beta \text{College}_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \tag{C.1}$$

where w_{it} is the log wage of individual i at time t, College is an indicator for whether individual i works in a college occupation, δ_i is an individual fixed effect, and X contains a cubic in potential experience, regional annual unemployment rate, aggregate annual unemployment rate, a quadratic in age, industrial (2-digit), regional, monthly, and yearly fixed effects. We follow Barnichon and Zylberberg (2019) in estimating equation (C.1) on "marginally" underemployed workers only, i.e., workers who transitioned from proper employment to underemployment and back to proper employment to control for selection based on unobservable characteristics into underemployment.

Table C.1 contains the estimated wage premium for college jobs. Each column represents a different combination of control variables and fixed effects. Column (4) represents our preferred specification that is used to calibrate the model in Section 5.

Table C.1: The Wage Premium of College Jobs

	(1)	(2)	(3)	(4)
College	0.3849***	0.3857***	0.2600***	0.2597***
	(0.0192)	(0.0196)	(0.0218)	(0.0223)
Exp	-0.0087***	-0.0026	0.0013	-0.0062***
	(0.0023)	(0.0020)	(0.0019)	(0.0021)
Exp^2	0.0002***	0.0000	-0.0000	0.0001**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	-0.0000	0.0000	0.0000**	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annual Urate				-0.0179**
				(0.0070)
Annual Urate				-0.0083
				(0.0064)
Age				0.4603***
				(0.0920)
Age^2				-0.0084***
				(0.0018)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,085	10,988	10,988	10,988
R^2	0.843	0.853	0.894	0.894

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of College. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

Table C.2 contains the estimation of wage premium by estimating equation (C.1) for college jobs identified by the ONET42.27 definition. Column (4) represents our preferred specification that is used to calibrate the ONET42.27 model in Section B.3.1.

Table C.2: The Wage Premium of College Jobs (ONET42.27 Definition)

	(1)	(2)	(3)	(4)
College	0.3382***	0.3390***	0.2409***	0.2426***
	(0.0184)	(0.0189)	(0.0228)	(0.0230)
Exp	-0.0081***	-0.0017	0.0010	-0.0102***
	(0.0025)	(0.0021)	(0.0020)	(0.0022)
Exp^2	0.0002*	-0.0000	-0.0001*	0.0001**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	-0.0000	0.0000*	0.0000***	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annaul Urate				-0.0110
				(0.0070)
Annaul Urate				-0.0070
				(0.0067)
Age				0.8568***
				(0.1308)
Age^2				-0.0159***
				(0.0026)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,209	11,112	11,112	11,112
R^2	0.823	0.833	0.883	0.884

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of College. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

Table C.3 contains the estimation of wage premium by estimating equation (C.1) for college jobs identified by the OOH2012 definition. Column (4) represents our preferred specification that is used to calibrate the OOH2012 model in Section B.3.1.

Table C.3: The Wage Premium of College Jobs (OOH2012 Definition)

	(1)	(2)	(3)	(4)
College	0.4182***	0.4076***	0.2766***	0.2747***
	(0.0207)	(0.0207)	(0.0208)	(0.0212)
Exp	0.0007	0.0078***	0.0108***	0.0033
	(0.0023)	(0.0020)	(0.0021)	(0.0022)
Exp^2	-0.0000	-0.0002***	-0.0002***	-0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	0.0000	0.0000***	0.0000***	0.0000***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annaul Urate				-0.0115*
				(0.0065)
Annaul Urate				0.0002
				(0.0058)
Age				0.5330***
				(0.1016)
Age^2				-0.0096***
				(0.0020)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,096	10,993	10,993	10,993
R^2	0.845	0.854	0.896	0.896

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of College. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

C.1.2 Unemployment and Underemployment Rates

The calculation of the unemployment and underemployment rates involves a three-step procedure. First, we count the number of individuals in the categories of unemployment, underemployment, and proper employment in each week. Second, the proportion of each labor status category is calculated by dividing the headcount of each category by the total headcount observed during that week. Finally, both an unweighted average and a weighted average are computed across all weeks, with the number of observations in that week serving as the weighting factor. Table C.4 contains the results for each definition of college jobs.

Table C.4: Composition of Labor Force Statuses

	Unemployed	Underemployed	Properly employed	NILF
ONET50.00				
Unweighted	0.032	0.408	0.497	0.063
Weighted	0.027	0.416	0.503	0.054
ONET42.27				
Unweighted	0.032	0.385	0.520	0.063
Weighted	0.027	0.388	0.531	0.054
OOH2012				
Unweighted	0.032	0.435	0.470	0.063
Weighted	0.027	0.433	0.485	0.054

C.1.3 Average U2N Duration

To determine the average U2N duration, we compute the time it takes for the new entrant to secure their first non-college job. To be consistent with our model, we presume that each college graduate initially has a 4-week (or 1-month) period of unemployment. We then track the number of months each unemployed graduate spends before finding their first non-college job. Next, this duration is averaged across individuals who have ever experienced underemployment. Ultimately, we find the average U2N duration to be 2.147/2.147/2.111 months under the ONET50.00/ONET42.27/OOH2012 definition for college jobs.

C.2 Calibration with Optimal Weighting Matrix

C.2.1 Methodology

We use the Method of Simulated Moments (MSM) to estimate parameters ($\hat{\vartheta}$) by minimizing the weighted distance between empirical moments (m) and simulated moments ($\tilde{m}$). In the baseline calibration, we use a scaled identity matrix, I/m^2, to compute the weighted distance. This approach minimizes the fractional deviation of the simulated moment from its corresponding data moment by normalizing the scales or units of the data moments, which ensures that no single moment disproportionately impacts the estimation due to its scale. The efficiency of MSM estimates can be improved by choosing a more efficient weighting matrix W^* to minimize the variance of the MSM estimator. In particular, a more efficient weighting matrix W^* is the inverse variance-covariance matrix of data moments, denoted as $W^* = S_m^{-1}$, that places less weight on moments with greater variance.

To construct the optimal weighting matrix, we start by determining the variance-covariance matrix of data moments (S_m). This is obtained by generating bootstrap samples and computing the data moments in each sample. Specifically, by randomly selecting observations from our full sample with replacement, we create a bootstrap sample identical in size to the original dataset. This step is repeated $1,000$ times, and each iteration yields a new bootstrap sample. Within each of these samples, we compute the moments targeted in the calibration. After collecting all data moments computed in each bootstrap sample, we compute the variance-covariance matrix (S_m). In this matrix, the diagonal elements denote the variances of each data moment, while the off-diagonal elements reflect the covariances between different moments.

Finally, we re-estimate the unknown parameters ($\hat{\vartheta}^*$) by minimizing the weighted deviation of the model moments from their corresponding data moments, where the weighting matrix is given by the inverse variance-covariance matrix. That is,

$$\hat{\vartheta}^* = \text{argmin } (\tilde{m} - m)' W^* (\tilde{m} - m), \ W^* = S_m^{-1}. \tag{C.2}$$

C.2.2 Comparison with the Baseline Calibration

The model fit using the optimal weighting matrix, and its comparison with the baseline calibration, are depicted in Figure C.1 and Table C.5. It is evident that the optimal weighting matrix does not significantly improve the model fit compared to the baseline calibration. The parameters calibrated using the optimal weighting matrix are detailed in Table C.6. Notably, the parameters calibrated under the optimal weighting matrix are not very different from those in the baseline calibration. Consequently, we adopt the model calibrated with the scaled identity matrix for the quantitative analysis in the main text.

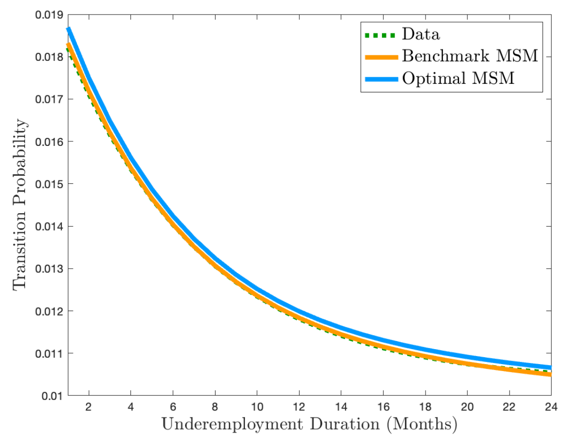


Figure C.1: Comparison between Baseline and Optimal MSM - Transition Path

Table C.5: Comparison between Baseline and Optimal MSM - Other Moments

Moment	Target	$\tilde{m}(\vartheta)$	$\tilde{m}(\vartheta^*)$
Unemployment rate	0.081	0.081	0.081
Underemployment rate	0.416	0.414	0.411
U2N duration	2.147	2.111	2.110
College job premium	0.260	0.259	0.260
b/[Average labor productivity]	0.710	0.707	0.707
$\partial \log(w_n)/\partial \upsilon$	-0.014	-0.014	-0.014
$\partial \log(w_c)/\partial \upsilon$	-0.014	-0.014	-0.014
$\partial \log(w_n)/\partial \tau$	0.001	0.001	0.001
$\partial \log(w_c)/\partial \tau$	-0.001	-0.001	-0.001

Table C.6: Comparison between Baseline and Optimal MSM - Parameter Values

	Definition	$\hat{\vartheta}$	$\hat{\vartheta}^*$
β	Discount factor	0.996	0.996
δ	Entry/exit probability	0.011	0.011
g_c	College productivity	1.000	1.000
g_n	Non-college productivity	0.745	0.744
b	Utility while unemployed	0.611	0.611
k_n	Non-college vacancy cost	2.167	2.173
k_c	College vacancy cost	2.054	2.043
λ	Employed search intensity	0.851	0.857
a^L	Suitability pr.: type L	0.023	0.023
a^H	Suitability pr.: type H	0.354	0.356
π	Pr. of being a type H worker	0.049	0.049
ϕ	Pr. of regaining college skills	0.006	0.006
$d_{c,\upsilon}$	College skill loss: unemp.	-0.014	-0.014
$d_{c,\tau}$	College skill loss: underemp.	-0.001	-0.001
$d_{n,\upsilon}$	Non-college skill loss: unemp.	-0.014	-0.014
$d_{n,\tau}$	Growth of non-college skills	0.001	0.001

C.2.3 Over-identification Test

Given that the number of data moments ($p = 33$) is greater than the number of unknown parameters ($d = 14$), the model is over-identified. When the model is over-identified, some moment conditions will be different from zero, which allows us to assess how well the estimated model matches the data. Following Jalali et al. (2015), we quantify the significance of calibration error between the model moments ($\tilde{m}$) and data moments (m) by computing the following J-statistic:

$$J = (\tilde{m} - m)' W^* (\tilde{m} - m) \sim \chi^2_{p-d}, \; W^* = S_m^{-1}. \tag{C.3}$$

For the calibration with $W^* = S_m^{-1}$, the J-statistic stands at $1.673 < \chi^2_{p-d} = 36.191$, indicating that at a 99% confidence level, there is no statistical difference between the estimated model and the true data-generating process.

C.3 Extended Model with Output Difference across Suitability Types

C.3.1 Model

To support our identifying assumption, we extend the model to incorporate the output in college jobs as a function of the worker's suitability. Specifically, we assume:

$$y_c^H(\upsilon,\tau) = \alpha y_c^L(\upsilon,\tau). \tag{C.4}$$

As the only modification to the model is the production technology in college jobs, we use this section to present the value of a match between a worker and a college job.

Consider a worker with history (υ,τ) and expected suitability μ who forms a new match with a college job. The expected output of the match is $\mu y_c^H(\upsilon,\tau) + (1-\mu)y_c^L(\upsilon,\tau)$. After producing for one period, the worker and firm can learn the worker's suitability type, as the match either produces $y_c^H(\upsilon,\tau)$ or $y_c^L(\upsilon,\tau)$ units of output. In the former (latter) case, the worker and firm learn that the worker has broad (limited) suitability. Let $V_{e,c}^i(\upsilon,\tau)$ represent the total surplus of a match between a worker with history (υ,τ) and is known to be type-i. It follows that

$$V_{e,c}^i(\upsilon,\tau) = y_c^i(\upsilon,\tau) + \beta(1-\delta)\{\phi V_{e,c}^i(1,0) + (1-\phi)V_{e,c}^i(\upsilon,\tau)\}. \tag{C.5}$$

Equation (C.5) has the same interpretation as equation (8) in the main text, except that the match output is indexed by the worker's suitability type.

Now let $V_{e,c}(\upsilon,\tau,\mu)$ denote the surplus at the formation of a match between a college job and worker with history (υ, τ) and expected suitability μ. It follows that $V_{e,c}(\upsilon, \tau, \mu)$ satisfies

$$V_{e,c}(\upsilon, \tau, \mu) = \mu y_c^H(\upsilon, \tau) + (1 - \mu) y_c^L(\upsilon, \tau) + \beta(1 - \delta)\big\{\mu[\phi V_{e,c}^H(1, 0) + (1 - \phi)V_{e,c}^H(\upsilon, \tau)] + (1 - \mu)[\phi V_{e,c}^L(1, 0) + (1 - \phi)V_{e,c}^L(\upsilon, \tau)]\big\}, \quad \text{(C.6)}$$

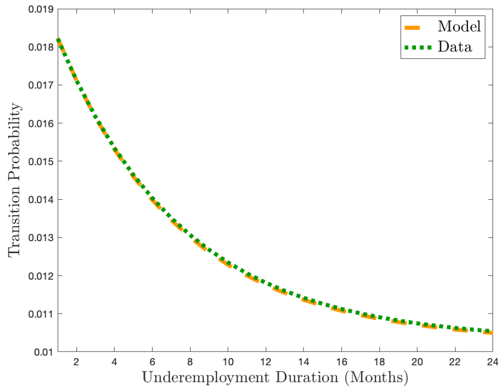
where $V_{e,c}^i(\upsilon, \tau)$ satisfies (C.5). From (C.6), a new match produces the expected output $\mu y_c^H(\upsilon, \tau) + (1 - \mu) y_c^L(\upsilon, \tau)$. With probability μ, the worker is a broad-suitable worker and therefore the match surplus in subsequent periods is determined by $V_{e,c}^H(\upsilon, \tau)$. With probability $1 - \mu$, the worker has a limited suitability and the match surplus in future periods is given by $V_{e,c}^L(\upsilon, \tau)$. Notice throughout equations (C.5) and (C.6) that we still account for the possibility of workers regaining their college skills.

With $V_{e,c}(\upsilon, \tau, \mu)$ in hand, one can compute the entry of firms into submarkets with college jobs using the entry condition, equation (9). The rest of the model's equilibrium conditions are unchanged relative to Section 4.

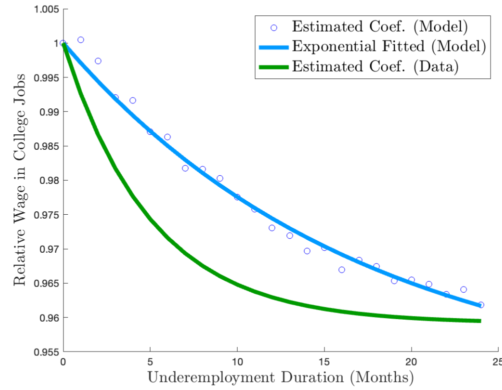
C.3.2 Calibration and Decomposition

This calibration approach is similar to the baseline model. However, instead of targeting the estimated wage effects from column (6) of Table 2, we target the relative wages in college jobs as a function of underemployment history that is generated by estimating the negative exponential model on college wages shown in Figure A.7(a). This is useful for calibrating α in equation (C.4), as the wage pattern, especially the degree of convexity in the wage decline in college jobs, is informative of the role of unobserved heterogeneity in determining output and wages in college jobs.

In a nutshell, we calibrate 15 parameters by targeting the transition path (24 moments), the path of college job wages (25 moments), as well as other 5 moments from the baseline calibration. Figure C.2 and Table C.7 present the fit of the extended model. The calibrated parameters are listed in Table C.8.



(a) Transition Path



(b) College Job Wage Path

Figure C.2: Model Fit

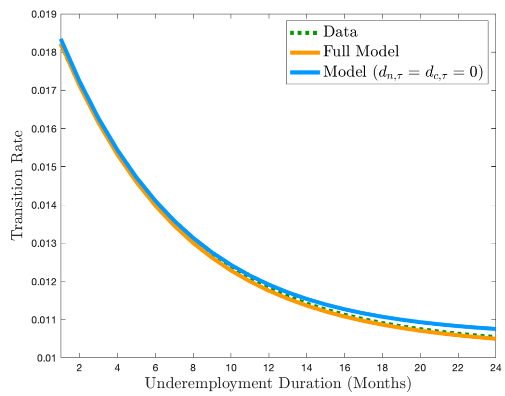
Table C.7: Model and Data Comparison

Moment	Target	Model
Unemployment rate	0.081	0.083
Underemployment rate	0.416	0.415
U2N duration	2.147	2.118
College job premium	0.260	0.273
b/[Average labor productivity]	0.710	0.707

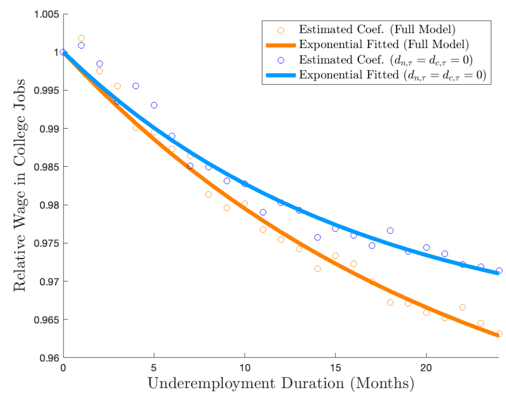
Table C.8: Parameter Values

	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.354
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.051
g_n	Non-college productivity	0.750	ϕ	Pr. of regaining college skills	0.006
b	Utility while unemployed	0.617	$d_{c,\upsilon}$	College skill loss: unemp.	-0.0142
k_n	Non-college vacancy cost	2.255	$d_{c,\tau}$	College skill loss: underemp.	-0.0004
k_c	College vacancy cost	2.046	$d_{n,\upsilon}$	Non-college skill loss: unemp.	-0.0138
λ	Employed search intensity	0.833	$d_{n,\tau}$	Growth of non-college skills	0.0006
α	Prod. of type-H workers	1.070	-	-	-

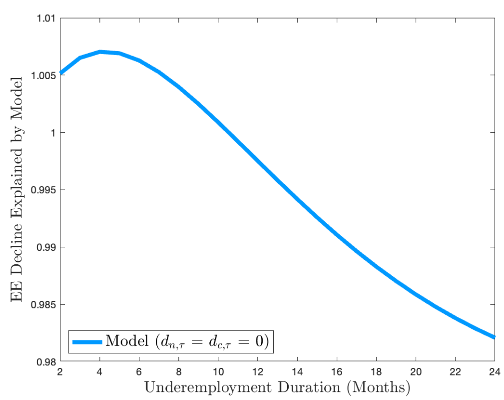
We proceed to the decomposition exercise presented in Section 5.2. Figure C.3 shows that, by deactivating skill accumulation and loss during underemployment, the transition probability for each underemployment history τ slightly increases. On the aggregate, the fraction explained by the model augmented with output differences across suitability types, becomes slightly larger as it introduces an additional channel through which selection contributes to the duration dependence of underemployment. Specifically, the model with only unobserved heterogeneity can explain 98.80% of the duration dependence observed in the data, which is slightly higher than what could be explained by the unobserved heterogeneity in the baseline model (95.27%).



(a) Transition Path



(b) Wage Path



(c) Fraction Explained at each τ

Figure C.3: Duration Dependence Decomposition

C.4 Robustness of Duration Dependence Decomposition

C.4.1 With Pre-set Skill Parameters

Instead of calibrating the skill accumulation and loss parameters to match the wage effects in column (6) of Table 2, we can instead rely on previous literature which evaluates the effect of nonemployment on measures of productivity and to set the skill parameters equal to what that literature has found. In other words, we can simply set the skill parameters so that they are in line with literature that evaluates the effect of nonemployment on productivity. To do this, we draw on the recent study by Dinerstein et al. (2022) who exploited quasi-random variation in teaching assignments in Greece to estimate the rate of skill depreciation and the returns to experience. While the setting is specific to teachers in Greece, this study provides what is arguably the best evidence to date on the effect of nonemployment and working on productivity at the individual level. They find a skill depreciation rate of 4.3% per year and a returns to experience of 6.8%. Therefore, the net effect of working on productivity is $6.8 - 4.3 = 2.5\%$ per year. With these estimates in mind, we set $d_{c,\tau} = -(.043)^{\frac{1}{12}} - 1 = -0.0035$ and $d_{n,\tau} = (0.025)^{\frac{1}{12}} - 1 = .0021$, which are simply monthly rates that correspond with the annual rates estimated by Dinerstein et al. (2022).[7] For the probability of college workers regaining their skills, we set ϕ so that the average increase in productivity after working in college jobs is 0.21% per month. Given that the magnitude of dynamics for non-college skills (0.0021) and college skills (-0.0035) during underemployment is nearly three to four times that of the estimated wage loss in the main text, which are 0.0006 and -0.0013 respectively, this exercise gives skill dynamics an opportunity to explain a larger proportion of the duration dependence in underemployment.

With the pre-set skill parameters in hand, we re-calibrate the model to target the transition path and the growth of college job wages, as well as the moments listed in Table 3, except for the four wage effect targets. Figure C.4 and Table C.9 present the fit of the

[7] Note that skill losses during unemployment are maintained at the same magnitude as in the baseline calibration.

model with pre-set skill parameters. Notably, the re-calibrated model fits the data well. The calibrated parameters are listed in Table C.10.

We also replicate the decomposition exercise presented in Section 5.2. Figure C.5 shows that by turning off skill accumulation and loss during underemployment, the transition probability at each underemployment history τ increases by a small amount. The model with unobserved heterogeneity explains 95.0% of the duration dependence in underemployment.

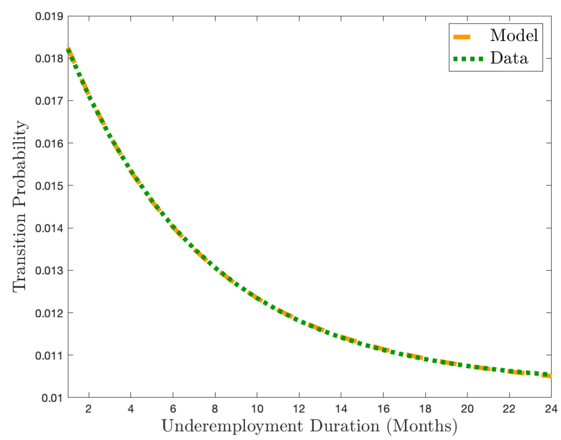


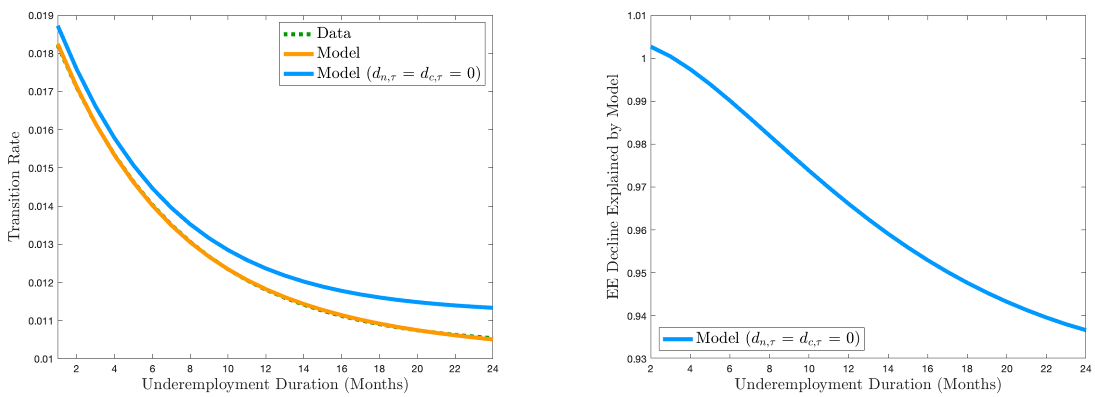
Figure C.4: Model Fit – Transition Path

Table C.9: Model and Data Comparison

Moment	Target	Model
Unemployment rate	0.081	0.081
Underemployment rate	0.416	0.416
U2N duration	2.147	2.105
College job premium	0.260	0.261
b/[Average labor productivity]	0.710	0.703
Recovery rate	0.0021	0.0021

Table C.10: Parameter Values

	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.350
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.049
g_n	Non-college productivity	0.727	ϕ	Pr. of regaining college skills	0.061
b	Utility while unemployed	0.617	$d_{c,\upsilon}$	College skill loss: unemp.	-0.0136
k_n	Non-college vacancy cost	1.605	$d_{c,\tau}$	College skill loss: underemp.	-0.0035
k_c	College vacancy cost	2.661	$d_{n,\upsilon}$	Non-college skill loss: unemp.	-0.0136
λ	Employed search intensity	0.915	$d_{n,\tau}$	Growth of non-college skills	0.0021



(a) Path of Transition Probabilities

(b) Fraction Explained at each τ

Figure C.5: Duration Dependence Decomposition

C.4.2 Effective Wages

Given the free entry condition, equation (9), the value of the worker's employment contract can be expressed as

$$x(\chi, \upsilon, \tau, \theta) = V_{e,\chi}(\upsilon, \tau) - \frac{k_\chi}{q(\theta_{\chi,\upsilon,\tau})}. \tag{C.7}$$

Recall that a worker's wage is equated with the output of the match. Therefore, the worker earns the entire value of the match, $V_{e,\chi}(\upsilon, \tau)$. It follows from (C.7) that the fee paid by the worker to the firm upon the formation of the match is given by $k_\chi/q(\theta)$, i.e. the average recruiting costs incurred by the firm. To derive the worker's effective wage (wage net of a per-period fee paid to the firm), we first need to derive an explicit expression for the per-period fee paid by the worker. Note that the present discounted value of the per-period fee payments should sum up the aggregate fee, $k_\chi/q(\theta)$. Let $\tilde{\xi}_\chi(\upsilon,\tau)$ denote the per-period fee paid by the worker with characteristics (υ,τ) at the beginning of the match to a type χ firm. It is straightforward to show

$$\tilde{\xi}_\chi(\upsilon,\tau) = \begin{cases} \frac{k_c[1-\beta(1-\delta)]}{q(\theta(n,\upsilon,\tau,x))} & \text{if } \chi = c, \\ \frac{k_n[q(\theta(c,\upsilon,\tau,x))]^{-1}}{1+\sum_{\tau=1}^{\bar{\tau}-1}[\beta(1-\delta)]^\tau \prod_{k=1}^{\tau}[1-\mu_k \lambda p(\theta(c,\upsilon,k,x)))]+\frac{[\beta(1-\delta)]^{\bar{\tau}} \prod_{k=1}^{\bar{\tau}}[1-\mu_k \lambda p(\theta(c,\upsilon,k,x))]}{1-\beta(1-\delta)[1-\mu_{\bar{\tau}} \lambda p(\theta(c,\upsilon,\bar{\tau},x))]}} & \text{if } \chi = n, \end{cases} \tag{C.8}$$

where the second line of (C.8), the fee paid to non-college firms, accounts for the chance that the worker transitions to a college job. The effective wage is given by $\tilde{w}_\chi(\upsilon,\tau) = y_\chi(\upsilon,\tau) - \tilde{\xi}_\chi(\upsilon,\tau)$.

Next, we re-calibrate the model with the same calibration strategy outlined in Section 5, except we use the effective wages. Figure C.6 and Table C.11 show that the model aligns closely with the data, while Table C.12 presents the parameter values.

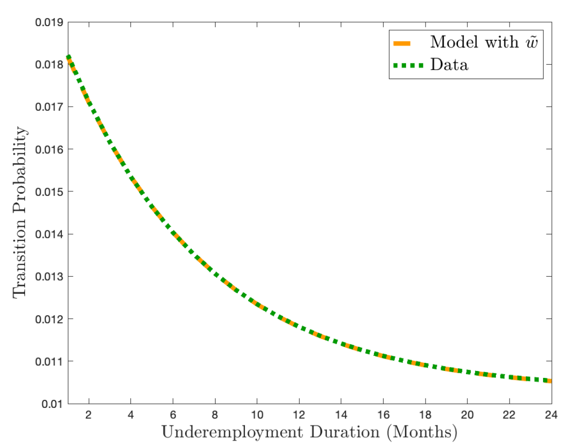


Figure C.6: Model Fit – Transition Path

To support our identification strategy, Figure C.7 shows that the effective wages in non-college jobs are responsive to the skill loss and accumulation parameters $d_{n,\upsilon}$ and $d_{n,\tau}$ while

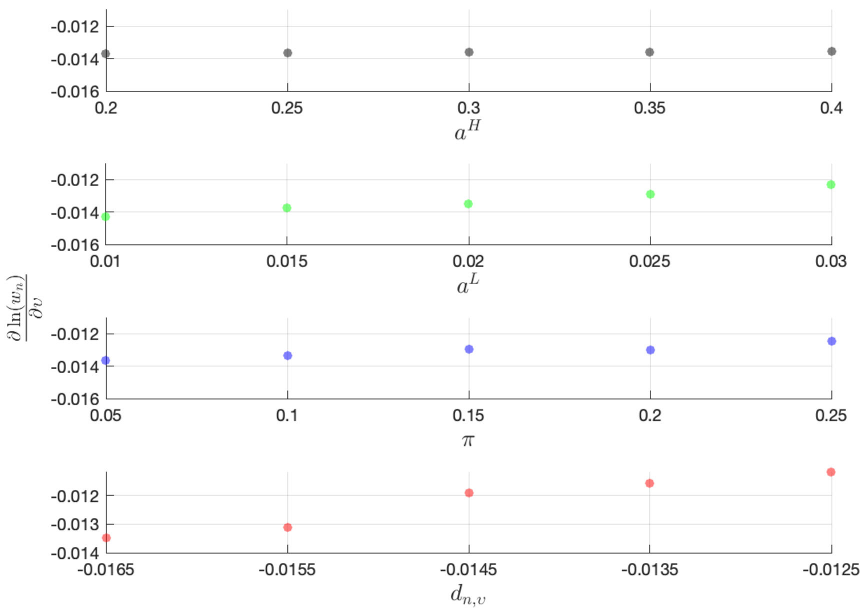
Table C.11: Model and Data Comparison

Moment	Target	Model	Moment	Target	Model
Unemployment rate	0.081	0.081	$\partial \log(w_n)/\partial \upsilon$	-0.014	-0.014
Underemployment rate	0.416	0.413	$\partial \log(w_c)/\partial \upsilon$	-0.014	-0.014
U2N duration	2.147	2.124	$\partial \log(w_n)/\partial \tau$	0.001	0.001
College job premium	0.260	0.258	$\partial \log(w_c)/\partial \tau$	-0.001	-0.001
b/[Average labor productivity]	0.710	0.710	-	-	-

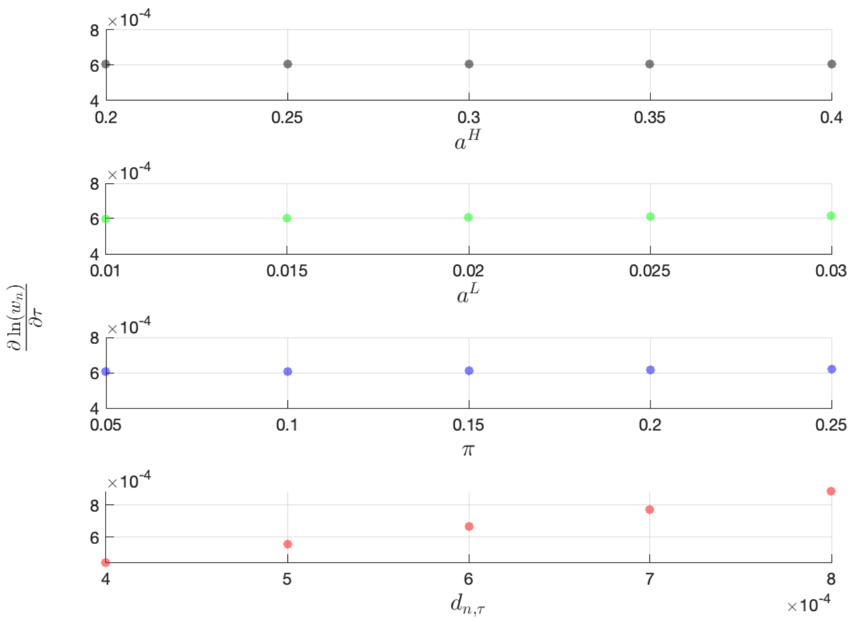
not being responsive to the unobserved heterogeneity parameters. Figure C.8 shows the same, but for effective wages in college jobs. Moreover, Figure C.9 shows that the path of transition probabilities is responsive to changes in the unobserved heterogeneity parameters, while Figure C.10 demonstrates that changes in the skill loss and accumulation parameters have little effect on the transition path.

Table C.12: Parameter Values

	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.019
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.289
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.054
g_n	Non-college productivity	0.804	ϕ	Pr. of regaining college skills	0.070
b	Utility while unemployed	0.644	$d_{c,\upsilon}$	College skill loss: unemp.	-0.012
k_n	Non-college vacancy cost	1.886	$d_{c,\tau}$	College skill loss: underemp.	-0.001
k_c	College vacancy cost	0.861	$d_{n,\upsilon}$	Non-college skill loss: unemp.	-0.014
λ	Employed search intensity	0.818	$d_{n,\tau}$	Growth of non-college skills	0.001

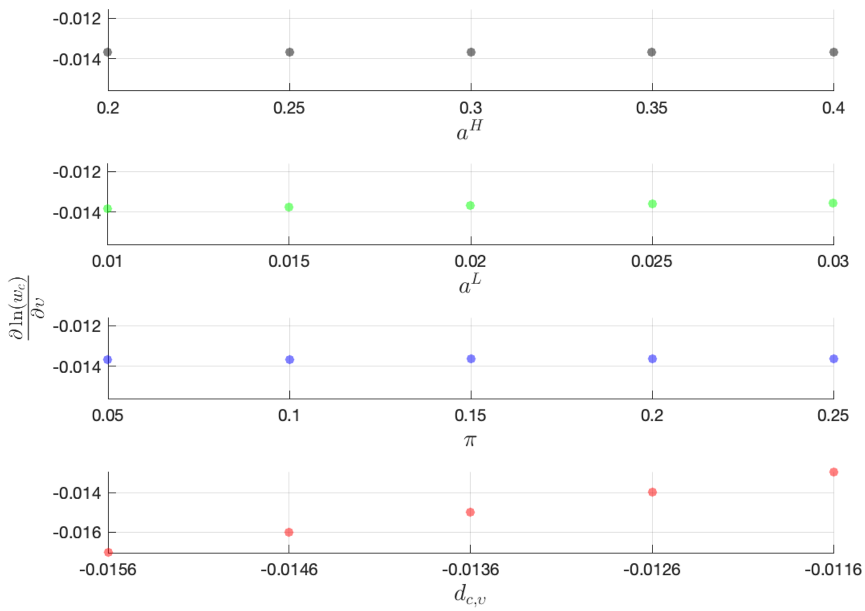


(a) $\partial \log(w_n)/\partial \upsilon$

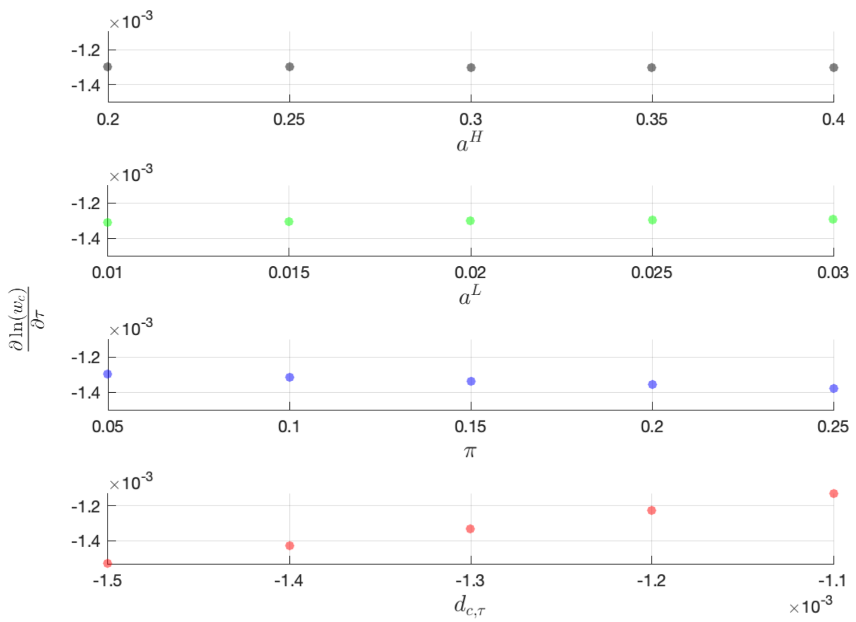


(b) $\partial \log(w_n)/\partial \tau$

Figure C.7: Comparative Statics of Wage Effects in Non-college Jobs

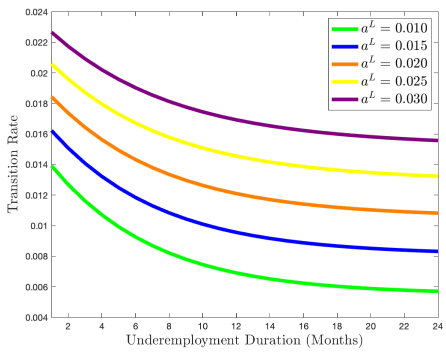


(a) $\partial \log(w_c)/\partial \upsilon$

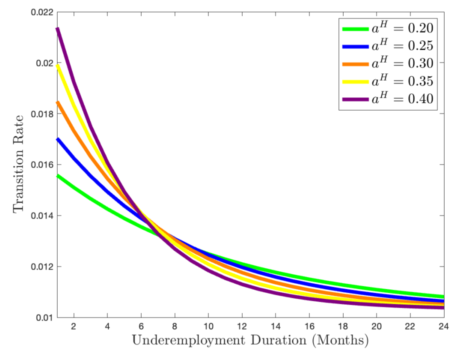


(b) $\partial \log(w_c)/\partial \tau$

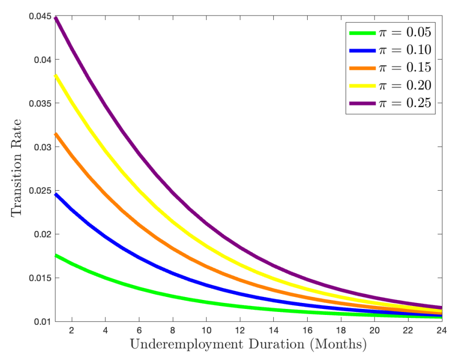
Figure C.8: Comparative Statics of Wage Effects in College Jobs



(a) Transition Path and a^L

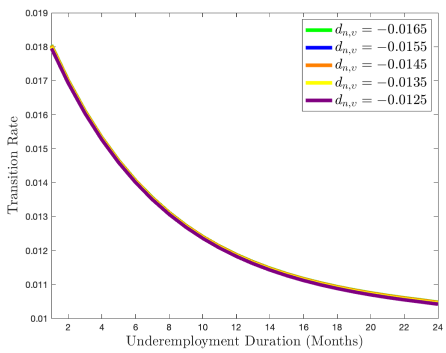


(b) Transition Path and a^H

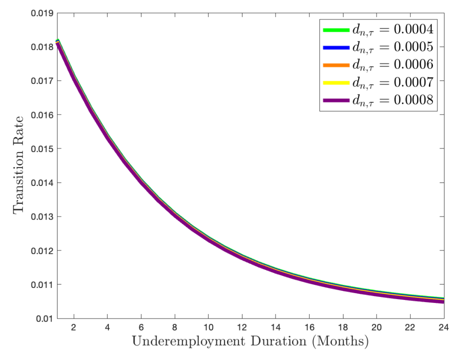


(c) Transition Path and π

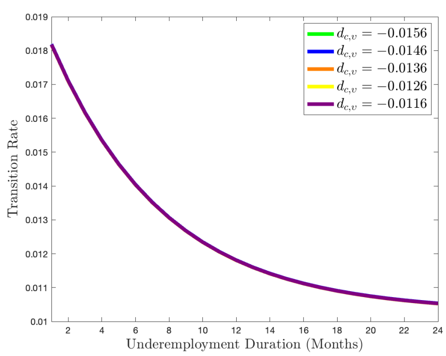
Figure C.9: The Transition Path and Unobserved Heterogeneity Parameters



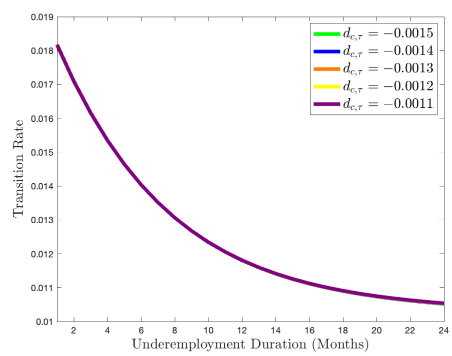
(a) Transition Path and $d_{n,\upsilon}$



(b) Transition Path and $d_{n,\tau}$



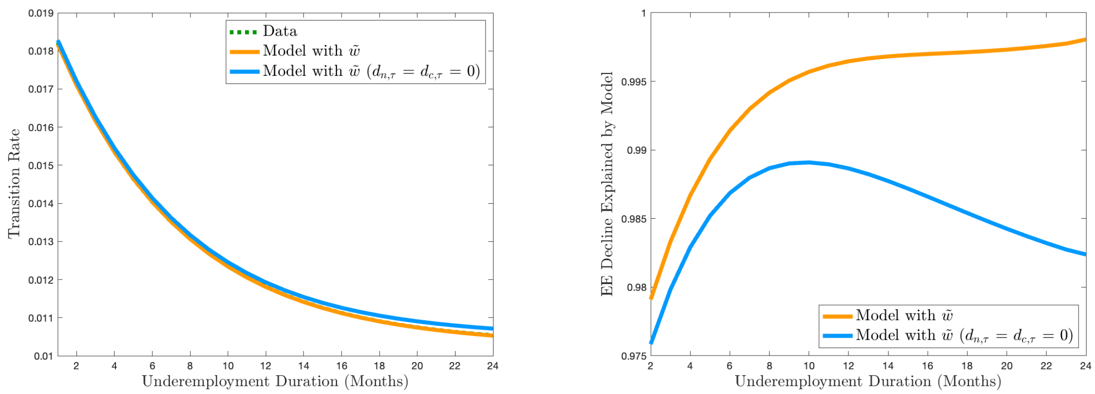
(c) Transition Path and $d_{c,\upsilon}$



(d) Transition Path and $d_{c,\tau}$

Figure C.10: The Transition Path and Skill Growth/Decay Parameters

Next, we turn off the accumulation and loss of skills during underemployment to assess how much of the duration dependence observed in the data can be explained by unobserved heterogeneity. The findings, as depicted in Figure C.11, reveal that shutting off skill dynamics during underemployment marginally alters the transition path. As for an aggregate decomposition, we find that 98.36% of the duration dependence is accounted for by unobserved heterogeneity.



(a) Path of Transition Probabilities

(b) Fraction Explained at each τ

Figure C.11: Duration Dependence Decomposition

D Theoretical Appendix

D.1 Laws of Motion

Let $u(\upsilon)$ denote the measure of workers who begin the period unemployed with unemployment history υ. The law of motion for unemployed workers is given by

$$\hat{u}(\upsilon) = \begin{cases} (1-\delta)\delta & \text{for } \upsilon = 1, \\ (1-\delta)u(\upsilon_-)[\varrho_{n,\upsilon_-}(1-p(\theta^*_{n,\upsilon_-})) + \varrho_{c,\upsilon_-}(1-\mu_{\upsilon_-}p(\theta^*_{c,\upsilon_-}))] & \text{for } \upsilon \in \{2,\ldots,\bar{\upsilon}-1\}, \\ (1-\delta)\sum_{\upsilon=\bar{\upsilon}}^{\bar{\upsilon}+1} u(\upsilon_-)[\varrho_{n,\upsilon_-}(1-p(\theta^*_{n,\upsilon_-})) + \varrho_{c,\upsilon_-}(1-\mu_{\upsilon_-}p(\theta^*_{c,\upsilon_-}))] & \text{for } \upsilon = \bar{\upsilon}, \end{cases} \tag{D.1}$$

where $\hat{u}(\upsilon)$ is the measure of unemployed workers with unemployment history υ at the beginning of the next period, $\upsilon_- \equiv \upsilon - 1$, $\varrho_{\chi,\upsilon} \in [0,1]$ is the fraction of unemployed workers with unemployment history υ who search for type χ jobs, μ_υ is the expected suitability of an unemployed worker with unemployment history υ, and $\theta^*_{\chi,\upsilon}$ is tightness associated with the policy function of unemployed workers with unemployment history υ who search for type χ jobs. From (D.1), the measure of unemployed workers who begin the next period unemployed with history $\upsilon \in \{2,\ldots,\bar{\upsilon}\}$ is given by those who began the previous period unemployed and did not find a job or exit the economy. For $\upsilon = 1$, the measure of unemployed workers is simply given by the new entrants to the labor market during stage 3 in the previous period who did not exit in stage 4.

Now let $e_\chi(\upsilon,\tau)$ denote the measure of workers with history (υ,τ) and are employed at type χ jobs at the beginning of the period. The law of motion for $e_n(\upsilon,\tau)$ is given by

$$\hat{e}_n(\upsilon,\tau) = \begin{cases} (1-\delta)u(\upsilon)\varrho_{n,\upsilon}p(\theta^*_{n,\upsilon}) & \text{for } \tau = 1, \\ (1-\delta)e_n(\upsilon,\tau_-)(1-\lambda\mu_{\upsilon,\tau_-}p(\theta^*_{c,\upsilon,\tau_-})) & \text{for } \tau \in \{2,\ldots,\bar{\tau}-1\}, \\ (1-\delta)\sum_{\tau=\bar{\tau}}^{\bar{\tau}+1} e_n(\upsilon,\tau_-)(1-\lambda\mu_{\upsilon,\tau_-}p(\theta^*_{c,\upsilon,\tau_-})) & \text{for } \tau = \bar{\tau}, \end{cases} \tag{D.2}$$

where $\mu_{\upsilon,\tau}$ is a worker with history (υ,τ)'s expected suitability, $\tau_- \equiv \tau - 1$, and $\theta^*_{\chi,\upsilon,\tau}$ is tightness associated with the policy function of an employed worker with history (υ,τ) in a submarket with type χ jobs. From (D.2), workers who begin the next period employed in non-college jobs and with $\tau = 1$ are comprised of unemployed workers who matched with a non-college job in the previous period. Workers who begin the next period with at least two periods of underemployment history are comprised of those who began the previous period underemployed and did not transition to a college job. All respective measures are multiplied by $(1-\delta)$ as this is the fraction of workers who remain in the labor market across periods.[8] The law of motion for $e_c(\upsilon,\tau)$ is given by

$$\hat{e}_c(\upsilon,\tau) = \begin{cases} (1-\delta)\big[u(\upsilon)\varrho_{c,\upsilon}\mu_\upsilon p(\theta^*_{c,\upsilon}) + e_c(\upsilon,\tau) + \phi(\sum_{\tau=1}^{\bar{\tau}}[e_c(\upsilon,\tau) + e_n(\upsilon,\tau)\lambda\mu_{\upsilon,\tau}p(\theta^*_{c,\upsilon,\tau})])\big] & \text{for } \tau = 0, \\ (1-\delta)(1-\phi)[e_c(\upsilon,\tau) + e_n(\upsilon,\tau)\lambda\mu_{\upsilon,\tau}p(\theta^*_{c,\upsilon,\tau})] & \text{for } \tau \in \{1,\ldots,\bar{\tau}\}. \end{cases} \tag{D.3}$$

The measure of workers who work in college jobs and have zero underemployment experience consists of unemployed workers who find a college job, those workers who are already employed in college jobs with $\tau = 0$, and finally a fraction ϕ of those employed in a college job with $\tau \geqslant 1$ or who transitioned from a non-college to college job and regained their college skills. The measure of workers employed in college jobs with $\tau > 0$ is given by those workers who began the previous period either already employed in a college job or transitioned from a non-college job to a college job and did not regain their college skills.

D.2 Propositions and Proofs

Proposition 1. *Consider a worker with history (υ,τ) and expected suitability μ who is currently employed in a type χ job. The worker will never search in a submarket for another type χ job. Moreover, if a worker is employed in a college job, then $\theta^*_{\chi,\upsilon,\tau} = 0$ for all $(\chi,\upsilon,\tau) \in X \times \Upsilon \times T$.*

[8] We have simplified equations (D.2)-(D.3) by accounting for the fact that workers employed in college jobs will not transition to a non-college job.

Proof. Suppose that a worker who is currently employed in a non-college job and searches in a submarket for a non-college job. Their submarket choice is given by

$$\theta = \arg\max\{-k_n\theta + \lambda p(\theta)(V_{e,n}(\upsilon, \hat{\tau}, \mu) - V_{e,n}(\upsilon, \hat{\tau}, \mu))\}. \tag{D.4}$$

Clearly the solution to (D.4) is $\theta = 0$.

For workers currently employed in college jobs and searching in a submarket for a college job, the worker's only potential benefit of finding a different college job is increasing or decreasing their expected suitability. Consider workers with an expected suitability $\mu = \mu^*$ for which $V_{e,c}(\upsilon, \tau, \mu)$ is maximized. For these workers, there is no benefit to finding another college job. Thus, they will not search for a different college job. Moreover, as we will show below, workers employed in a college job will never transition to a non-college job. Hence,

$$V_{e,c}(\upsilon, \tau, \mu^*) = \frac{y_c(\tau)}{1 - \beta(1-\delta)(1-\phi)} + \frac{\phi\beta(1-\delta)y_c(0)}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\phi)]}. \tag{D.5}$$

For workers with $\mu \neq \mu^*$, they can choose to never search for a college job, which would generate the same match value shown in equation (D.5). It follows that a worker employed in a college job would not search for another college job. More generally, workers employed in a type χ job will never transition to another type χ job.

Now suppose that the worker employed in a college job searches for a non-college job. Their submarket choice is

$$\theta = \arg\max\{-k_n\theta + \lambda p(\theta)(V_{e,n}(\upsilon, \tau, \mu) - V_{e,c}(\upsilon, \tau, \mu))\}. \tag{D.6}$$

A worker would not transition to a non-college job to only transition back to a college job in the future as underemployment leads to depreciation of college occupation-specific human capital. Therefore, if the worker transitions to a non-college job, they will remain in a non-college job until they exit the labor force. It follows that the sum of the worker's lifetime utility and firm's profits in a non-college job is bounded by

$$\bar{V}_{e,n}(\upsilon, \tau, \mu) = \frac{y_n(\bar{\tau})}{1 - \beta(1-\delta)}. \tag{D.7}$$

If, however, the worker were to remain employed in the college job until exiting the labor force, the value of their current employment relationship would be given by (D.5) with μ as the last argument in place of μ^*. Clearly $V_{e,c}(\upsilon, \tau, \mu) > \bar{V}_{e,n}(\upsilon, \tau, \mu)$ as we have assumed $y_c(\tau) > y_n(\tau)$ for all $\tau \in T$. Therefore, the solution to (D.6) is $\theta = 0$. ■

Proposition 2. *Assume that $a^H = a^L = 1$, which turns off the unobserved heterogeneity channel. Further, let $\Delta(\tau) = V_{e,c}(\tau) - V_{e,n}(\tau)$. Tightness, θ_τ, satisfies*

$$k_c \geqslant p'(\theta_\tau)\Delta(\tau), \tag{D.8}$$

where $\theta_\tau \geqslant 0$ with complementary slackness. We have the following results:

(i) $\Delta(\tau)$ *is strictly decreasing in* τ.

(ii) $\lambda p(\theta_\tau)$ *is weakly decreasing in* τ.

(iii) $\lambda p(\theta_\tau)$ *is generally concave in* τ.

Proof. We denote $V_{e,\chi}(\tau)$ as the sum of the worker's utility and firm's profits in a match between a type χ job and worker with underemployment history τ. It is straightforward to show:

$$V_{e,n}(\tau) = y_n(\tau) + \beta(1-\delta)\{V_{e,n}(\hat{\tau}) - \lambda k_c \hat{\theta} + \lambda p(\hat{\theta})\Delta(\hat{\tau})\}, \tag{D.9}$$

$$V_{e,c}(\tau) = \frac{y_c(\tau)[1-\beta(1-\delta)] + \phi\beta(1-\delta)y_c(0)}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\phi)]}, \tag{D.10}$$

where $\hat{\theta}$ solves

$$k_c \geqslant p'(\hat{\theta})\Delta(\hat{\tau}). \tag{D.11}$$

Part (i): We proceed via proof by contradiction. Suppose that $\Delta(\tau)$ is strictly increasing in τ. Consider $V_{e,n}(\bar{\tau})$ and $V_{e,n}(\bar{\tau}-1)$. It is easy to show that

$$V_{e,n}(\bar{\tau}) - V_{e,n}(\bar{\tau}-1) = y_n(\bar{\tau}) - y_n(\bar{\tau}-1) > 0. \tag{D.12}$$

Now consider

$$\begin{aligned} V_{e,n}(\bar{\tau}-1) - V_{e,n}(\bar{\tau}-2) &= y_n(\bar{\tau}-1) - y_n(\bar{\tau}-2) + \\ &\beta(1-\delta)\{V_{e,n}(\bar{\tau}) - V_{e,n}(\bar{\tau}-1) - k_c\theta^* + \lambda p(\theta^*)\Delta(\bar{\tau}) + k_c\theta^{**} - \lambda p(\theta^{**})\Delta(\bar{\tau}-1)\}, \end{aligned} \tag{D.13}$$

where θ^* and θ^{**}, respectively, solve

$$k_c \geqslant p'(\theta^*)\Delta(\bar{\tau}), \tag{D.14}$$

$$k_c \geqslant p'(\theta^{**})\Delta(\bar{\tau}-1). \tag{D.15}$$

From (D.13), $V_{e,n}(\bar{\tau}-1) - V_{e,n}(\bar{\tau}-2) > 0$ as $y_n(\bar{\tau}-1) > y_n(\bar{\tau}-2)$, $V_{e,n}(\bar{\tau}) > V_{e,n}(\bar{\tau}-1)$ from equation (D.12), and (assuming interior solutions), $-k_c\theta^* + \lambda p(\theta^*)\Delta(\bar{\tau}) > -k_c\theta^{**} + \lambda p(\theta^{**})\Delta(\bar{\tau}-1)$ as $\Delta(\bar{\tau}) > \Delta(\bar{\tau}-1)$ (by assumption) and $\theta^* > \theta^{**}$ following (D.14)-(D.15). We can extend this logic to show that $V_{e,n}(\tau) < V_{e,n}(\hat{\tau})$ for all $\tau \in \{1, 2, \ldots, \bar{\tau}-1\}$ and $\hat{\tau} = \min\{\tau+1, \bar{\tau}\}$. In other words, under the assumption that $\Delta(\tau)$ is increasing in τ, $V_{e,n}(\tau)$ is also increasing in τ. However, we can see from (D.10) that $V_{e,c}(\tau)$ is weakly decreasing in τ as $y_c(\tau)$ is weakly decreasing in τ. Hence, $\Delta(\tau) = V_{e,c}(\tau) - V_{e,n}(\tau)$ is decreasing in τ, which is a contradiction.

Part (ii): We now proceed to show that θ is weakly decreasing in τ. In the main text, we showed that the optimal choice of θ satisfies

$$k_c \geqslant p'(\theta)\Delta(\tau). \tag{D.16}$$

For this part of the proof, we assume an interior solution to (D.16). Following part (i), where we have shown $\Delta(\tau)$ is decreasing in τ, it follows that the optimal θ which satisfies equation (D.16) is decreasing in τ as $p(\theta)$ is strictly concave and, hence, $p'(\theta)$ is strictly decreasing in θ. As $p(\theta)$ is strictly increasing in θ, it follows that $\lambda p(\theta)$ is strictly decreasing in τ for all τ such that $\theta > 0$ satisfies (D.16). If $\theta = 0$ solves (D.16) for some $\tau^* \in T$, it follows that $\theta = 0$ for all $\tau \in \{\tau^*, \ldots, \bar{\tau}\}$ as, following part (i), $\Delta(\tau)$ is decreasing in τ. Hence, $\lambda p(\theta) = 0$ for all $\tau \in \{\tau^*, \ldots, \bar{\tau}\}$ and $\lambda p(\theta)$ is weakly decreasing in τ.

Part (iii): We show that $\lambda p(\theta)$ is generally concave in τ. Assuming an interior solution, θ_τ solves

$$p'(\theta_\tau) = \frac{k_c}{\Delta(\tau)}. \tag{D.17}$$

As $\Delta(\tau)$ is decreasing in τ (shown in part (i)), it follows that $k_c/\Delta(\tau)$ is increasing in τ. From (D.17), θ_τ is decreasing in τ, as $p'(\theta)$ is decreasing in θ. As for the concavity of $p(\theta_\tau)$, it is sufficient to characterize when the function $k_c/\Delta(\tau)$ is convex. To ease the exposition, suppose for the rest of this proof that $\tau \in \mathbb{R}_+$ and that $\Delta(\tau)$ is a twice continuously differentiable function. Let $g(\tau) \equiv [\Delta(\tau)]^{-1}$. The second derivative of $g(\tau)$ is

$$g''(\tau) = \frac{-\Delta''(\tau)[\Delta(\tau)]^2 + 2\Delta'(\tau)\Delta(\tau)\Delta'(\tau)}{[\Delta(\tau)]^4}. \tag{D.18}$$

It follows that $g''(\tau) > 0$, and $k_c/\Delta(\tau)$ is convex, if and only if

$$\frac{2[\Delta'(\tau)]^2}{\Delta(\tau)} > \Delta''(\tau). \tag{D.19}$$

There are three cases to consider.

1. $\Delta''(\tau) = 0$, i.e., $\Delta(\tau)$ is linear. Then (D.19) is satisfied.

2. $\Delta''(\tau) < 0$, i.e., $\Delta(\tau)$ is concave. Then (D.19) is satisfied.

3. $\Delta''(\tau) > 0$, i.e., $\Delta(\tau)$ is convex. In general, (D.19) is not guaranteed to hold. However, as τ increases and $\Delta(\tau)$ approaches zero, the left side of (D.19) approaches infinity.

However, as τ increases, the right side of (D.19) decreases. Hence, (D.19) is more likely to be satisfied at higher values of τ in the case where $\Delta''(\tau) > 0$.

To summarize, part (iii) has shown that the function $k_c/\Delta(\tau)$ is generally convex in τ, especially at higher values of τ. It follows that, when $k_c/\Delta(\tau)$ is convex, θ_τ is concave in τ in order to satisfy (D.17) (through the concavity of $p(\cdot)$). If θ_τ is concave, then $p(\theta_\tau)$ is also concave. ∎

Proposition 3. *Consider the worker's expected suitability at underemployment duration, τ, for $\tau \in \{1, 2, \dots, \bar{\tau}\}$:*

$$\mu_\tau = a^H - \frac{(a^H - \mu_{\tau-1})(1 - pa^L)}{1 - p\mu_{\tau-1}}. \tag{D.20}$$

Suppose that the matching probability of a suitable worker, p, is independent of τ and $p > 0$. We have the following results:

(i) $\mu_\tau = \mu_{\tau-1}$ *if* $\mu_{\tau-1} \in \{a^L, a^H\}$.

(ii) If $a^L < \mu_{\tau-1} < a^H$, *then* $\mu_\tau < \mu_{\tau-1}$.

(iii) If $\mu_{\tau-1} < 0.5[a^L + a^H]$, *then* $\partial[\mu_\tau - \mu_{\tau-1}]/\partial\mu_{\tau-1} < 0$.

(iv) Let $a^L = \alpha a^H$ *where* $\alpha \in [0, 1)$ *and* $a^L < \mu_{\tau-1} < a^H$. $\partial[\mu_\tau - \mu_{\tau-1}]/\partial\alpha > 0$.

Proof. Part (i): Substituting $\mu_{\tau-1} = a^L$ into (D.20) gives $\mu_\tau = \mu_{\tau-1} = a^L$. Through the same process, we have $\mu_\tau = \mu_{\tau-1}$ if $\mu_{\tau-1} = a^H$.

Part (ii): Taking the difference between μ_τ and $\mu_{\tau-1}$ gives

$$\mu_\tau - \mu_{\tau-1} = \frac{p(a^H - \mu_{\tau-1})(a^L - \mu_{\tau-1})}{1 - p\mu_{\tau-1}} < 0, \tag{D.21}$$

as $a^L < \mu_{\tau-1} < a^H$. Hence, $\mu_\tau < \mu_{\tau-1}$.

Part (iii): Differentiating (D.21) with respect to $\mu_{\tau-1}$ gives

$$\frac{\partial[\mu_\tau - \mu_{\tau-1}]}{\partial\mu_{\tau-1}} = \frac{p(2\mu_{\tau-1} - a^H - a^L)(1 - p\mu_{\tau-1}) + p^2(a^H - \mu_{\tau-1})(a^L - \mu_{\tau-1})}{(1 - p\mu_{\tau-1})^2}. \tag{D.22}$$

As $a^L < \mu_{\tau-1} < a^H$, it follows that $p^2(a^H - \mu_{\tau-1})(a^L - \mu_{\tau-1}) < 0$. Moreover, $p\mu_{\tau-1} < 1$. Thus, a sufficient condition for $\partial[\mu_\tau - \mu_{\tau-1}]/\partial\mu_{\tau-1} < 0$ is $2\mu_{\tau-1} - a^H - a^L < 0$, or $\mu_{\tau-1} < 0.5 * [a^H + a^L]$.

Part (iv): Replacing a^L with αa^H in equation (D.21) gives

$$\mu_\tau - \mu_{\tau-1} = \frac{p(a^H - \mu_{\tau-1})(\alpha a^H - \mu_{\tau-1})}{1 - p\mu_{\tau-1}}. \tag{D.23}$$

Hence,

$$\frac{\partial[\mu_\tau - \mu_{\tau-1}]}{\partial\alpha} = \frac{a^H p(a^H - \mu_{\tau-1})}{1 - p\mu_{\tau-1}} > 0, \tag{D.24}$$

as $\mu_{\tau-1} < a^H$ and $p\mu_{\tau-1} < 1$. ■

E Model with Full Information

This appendix provides further details on the model with full information referenced in Section 5.2. For brevity, we only present the details in the environment and equilibrium that are new relative to the baseline model presented in Section 3.

E.1 Environment

Workers learn their suitability type upon entering the labor market. A worker's suitability type is public information. The labor market continues to be organized in a continuum of submarkets. In the full information case, however, submarkets are also indexed by the worker's suitability type. Denoting $A = \{L, H\}$ and an individual's worker suitability type by i, the labor market is now organized in a continuum of submarkets indexed by $\omega = (\chi, i, \upsilon, \tau, x) \in X \times A \times \Upsilon \times T \times \mathbb{R}$. That is, in submarket ω, type χ firms search for a type-i worker with labor market history (υ, τ) and offer suitable workers an employment contract worth x in lifetime utility.

E.2 Value Functions

The value functions in the full information version of the model are very similar to those in the baseline model. The main exception is that the probability of being suitable for a college job, a^i, takes the place of, μ, the expected suitability in the version with information frictions. Here is the value of an unemployed worker of suitability type i who searches for a non-college job:

$$V_{u,n}(\upsilon, i) = b + \beta(1-\delta)\{V_u(\hat{\upsilon}, i) + R_n(x, V_u(\hat{\upsilon}, i))\}, \tag{E.1}$$

where

$$V_u(\upsilon, i) = \max\{V_{u,n}(\upsilon, i), V_{u,c}(\upsilon, i)\} \tag{E.2}$$

is the value of unemployment for a type-i worker with unemployment history υ and

$$R_\chi(x, V_u(\hat{\upsilon}, i)) = \max_x p(\theta(\chi, i, \hat{\upsilon}, 0, x))(x - V_u(\hat{\upsilon}, i)). \tag{E.3}$$

The value of searching for a college job satisfies:

$$V_{u,c}(\upsilon, i) = b + \beta(1-\delta)\{V_u(\hat{\upsilon}, i) + a^i R_c(x, V_u(\hat{\upsilon}, i))\}. \tag{E.4}$$

The sum of the worker's lifetime utility and firm's profits in a match between a non-college job and type-i worker with history (υ, τ) is given by:

$$V_{e,n}(\upsilon, \tau, i) = y_n(\upsilon, \tau) + \beta(1-\delta)\{V_{e,n}(\upsilon, \hat{\tau}, i) + \lambda a^i S(\upsilon, \hat{\tau}, i)\}, \tag{E.5}$$

where

$$S(\upsilon, \hat{\tau}, i) = \max_x p(\theta(c, i, \upsilon, \hat{\tau}, x))(x - V_{e,n}(\upsilon, \hat{\tau}, i)). \tag{E.6}$$

Finally, sum of the worker's lifetime utility and the firm's profits in a match between a college job and a type-i worker with history (υ, τ), $V_{e,c}(\upsilon, \tau, i)$, satisfies

$$V_{e,c}(\upsilon, \tau, i) = y_c(\upsilon, \tau) + \beta(1-\delta)\{\phi V_{e,c}(1, 0, i) + (1-\phi) V_{e,c}(\upsilon, \tau, i)\}. \tag{E.7}$$

E.3 Free Entry

In any submarket visited by a positive number of workers, tightness is consistent with the firm's incentives to create vacancies if and only if

$$k_\chi \geqslant q(\theta(\chi, i, \upsilon, \tau, x))\{V_{e,\chi}(\upsilon, \tau, i) - x\}, \tag{E.8}$$

and $\theta(\chi, i, \upsilon, \tau, x) \geqslant 0$ with complementary slackness. We restrict attention to equilibria in which $\theta(\chi, i, \upsilon, \tau, x)$ satisfies the complementary slackness condition in every submarket, even those that are not visited by workers.

E.4 Laws of Motion

Let $u_i(\upsilon)$ denote the measure of workers of suitability type i who begin the period unemployed with unemployment history υ. The law of motion is given by

$$\hat{u}_i(\upsilon) = \begin{cases} (1-\delta)\delta\pi^i & \text{for } \upsilon = 1, \\ (1-\delta)u_i(\upsilon_-)[\varrho_{i,n,\upsilon_-}(1-p(\theta^*_{i,n,\upsilon_-})) + \varrho_{i,c,\upsilon_-}(1-a^i p(\theta^*_{i,c,\upsilon_-}))] & \text{for } \upsilon \in \{2,\ldots,\bar{\upsilon}-1\}, \\ (1-\delta)\sum\limits_{\upsilon=\bar{\upsilon}}^{\bar{\upsilon}+1} u_i(\upsilon_-)[\varrho_{i,n,\upsilon_-}(1-p(\theta^*_{i,n,\upsilon_-})) + \varrho_{i,c,\upsilon_-}(1-a^i p(\theta^*_{i,c,\upsilon_-}))] & \text{for } \upsilon = \bar{\upsilon}, \end{cases} \tag{E.9}$$

where $\pi^H = \pi$, $\pi^L = 1-\pi$, $\hat{u}_i(\upsilon)$ is the measure of unemployed workers with unemployment history υ and suitability type i at the beginning of the next period, $\upsilon_- \equiv \upsilon - 1$, $\varrho_{i,\chi,\upsilon} \in [0,1]$ is the fraction of unemployed workers with suitability type i and unemployment history υ who search for type χ jobs, and $\theta^*_{i,\chi,\upsilon}$ denotes tightness associated with the policy function of unemployed workers with suitability type i and unemployment history υ who search for type χ jobs.

Now let $e_{i,\chi}(\upsilon,\tau)$ denote the measure of workers with suitability type i and history (υ,τ) who are employed at type χ jobs at the beginning of the period. The law of motion for $e_{i,n}(\upsilon,\tau)$ is given by

$$\hat{e}_{i,n}(\upsilon,\tau) = \begin{cases} (1-\delta)\varrho_{i,n,\upsilon}u_i(\upsilon)p(\theta^*_{i,n,\upsilon}) & \text{for } \tau = 1, \\ (1-\delta)e_{i,n}(\upsilon,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,\upsilon,\tau_-})) & \text{for } \tau \in \{2,\ldots,\bar{\tau}-1\}, \\ (1-\delta)\sum_{\tau=\bar{\tau}}^{\bar{\tau}+1} e_{i,n}(\upsilon,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,\upsilon,\tau_-})) & \text{for } \tau = \bar{\tau}, \end{cases} \tag{E.10}$$

where $\tau_- \equiv \tau - 1$, $\theta^*_{i,\chi,\upsilon,\tau}$ is tightness associated with the policy function of an employed worker with suitability type i and history (υ,τ) in a submarket with type χ jobs.

The law of motion for $e_{i,c}(\upsilon,\tau)$ is given by

$$\hat{e}_{i,c}(\upsilon,\tau)=\begin{cases}(1-\delta)[e_{i,c}(\upsilon,\tau)+u_i(\upsilon)\varrho_{i,c,\upsilon}a^i p(\theta^*_{i,c,\upsilon})+\phi(e_{i,c}-e_{i,c}(1,0)+e^*_{i,n}+u^*_{i,c})] & \text{for } \upsilon=1 \text{ and } \tau=0,\\ (1-\delta)(1-\phi)[u_i(\upsilon)\varrho_{i,c,\upsilon}a^i p(\theta^*_{i,c,\upsilon})+e_{i,c}(\upsilon,\tau)] & \text{for } \upsilon\geqslant 2 \text{ and } \tau=0,\\ (1-\delta)(1-\phi)[e_{i,n}(\upsilon,\tau)\lambda a^i p(\theta^*_{i,c,\upsilon,\tau})+e_{i,c}(\upsilon,\tau)] & \text{for } \upsilon\geqslant 2 \text{ and } \tau\geqslant 1,\end{cases} \tag{E.11}$$

where $e_{i,c}=\sum_{\upsilon\in\Upsilon}\sum_{\tau\in T}e_{i,c}(\upsilon,\tau)$ is the total measure of type-i workers employed in college jobs at the beginning of a period, $e^*_{i,n}=\lambda\sum_{\upsilon\in\Upsilon}\sum_{\tau\in T}e_{i,n}(\upsilon,\tau)a^i p(\theta^*_{i,c,\upsilon,\tau})$ is the total measure of type-i workers who transitioned from a non-college to college job within the period, and $u^*_{i,c}=a^i\sum_{\upsilon=2}^{\bar{\upsilon}}u_i(\upsilon)\varrho_{i,c,\upsilon}p(\theta^*_{i,c,\upsilon})$ is the total measure of unemployed workers with unemployment history $\upsilon\in\{2,\ldots,\bar{\upsilon}\}$ who found a college job in the previous period.

E.5 Equilibrium Definition

Definition 1. A stationary recursive equilibrium consists of a market tightness function $\theta(\omega)\colon X\times A\times\Upsilon\times T\times\mathbb{R}\to\mathbb{R}_+$, a value function for unemployed workers, $V_u(\upsilon,i)\colon\Upsilon\times A\to\mathbb{R}$, a policy function for unemployed workers, $\omega^*_u(\upsilon,i)\colon\Upsilon\times A\to X\times\mathbb{R}$, a joint value function for the worker-firm match, $V_{e,\chi}(\upsilon,\tau,i): X\times\Upsilon\times T\times A\to\mathbb{R}$, a policy function for the worker-firm match, $\omega^*_{e,\chi}(\upsilon,\tau,i): X\times\Upsilon\times T\times A\to X\times\mathbb{R}$, and a distribution of workers across the states of employment. The functions satisfy the following conditions. First, $\theta(\omega)$ satisfies (E.8) and the slackness condition for all $\omega\in X\times A\times\Upsilon\times T\times\mathbb{R}$. Second, $V_u(\upsilon,i)$ satisfies (E.2) for all $(\upsilon,i)\in\Upsilon\times A$ and $\omega^*_u(\upsilon,i)$ is the associated policy function. Third, $V_{e,n}(\upsilon,\tau,i)$ and $V_{e,c}(\upsilon,\tau,i)$ satisfy equations (E.5) and (E.7) for all $(\upsilon,\tau,i)\in\Upsilon\times T\times A$ and $\omega^*_{e,\chi}(\upsilon,\tau,i)$ is the associated policy function. Finally, the distribution of workers satisfies the laws of motion specified in Section E.4.

E.6 Quantitative Analysis

To take a closer look at the role of information friction in determining duration dependence, we assume the worker's suitability type is publicly observable. The first result that emerges from removing information frictions is that broad-suitable workers ($i = H$) never search for non-college jobs, as illustrated by the unemployed worker's policy function in Figure E.1. Consequently, the pool of underemployed workers consists solely of limited-suitability workers ($i = L$).

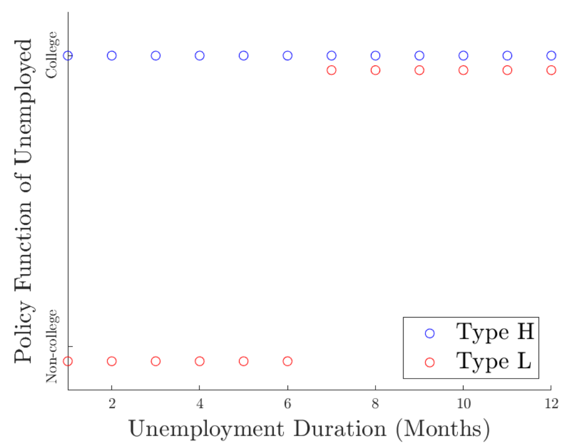
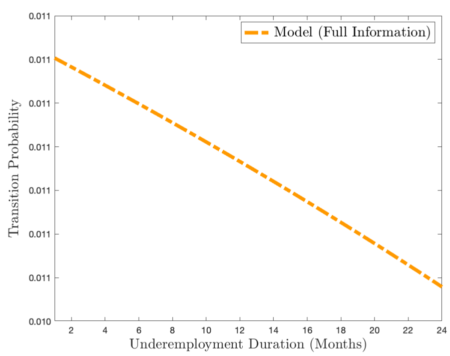
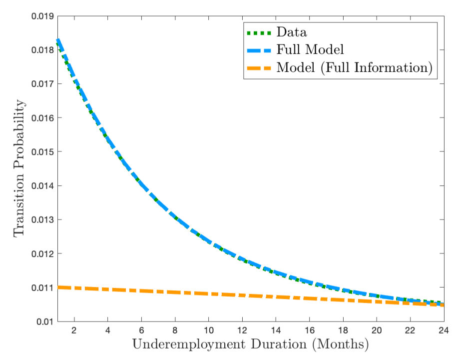


Figure E.1: Policy Function of the Unemployed

Figure E.2 shows that, with the pool of underemployed workers being exclusively composed of limited-suitability workers, a mild degree of negative duration dependence is still observed. Notably, the transition probability decreases from 0.01100 at $\tau = 1$ to 0.01048 at $\tau = 24$. The magnitude of this decline is negligible when compared to the full model.



(a) No Information Frictions



(b) Model and Data Comparisons

Figure E.2: Duration Dependence with and without Information Frictions

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