The Underemployment Trap*

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Abstract

Many college graduates are underemployed, i.e., work in occupations that do not require a college degree. We document that underemployed workers are less likely to transition to a college occupation the longer they are underemployed and that longer underemployment histories are associated with lower wages in college occupations. To explain these findings, we develop a directed search model with unobserved heterogeneity, occupation-specific human capital, and on the job search. Workers are uncertain about their suitability for college jobs and learn through search. Underemployment is generated by search and information frictions, as workers with a low expected job-finding probability in college occupations self-select into underemployment. Once underemployed, workers' college occupation-specific human capital decays. A quantitative decomposition shows that unobserved heterogeneity explains most of the duration dependence in underemployment, and that information frictions play a significant role in both the existence of underemployment and the resulting duration dependence.

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1 Introduction

A significant fraction of college graduates in the US are underemployed, i.e., work in jobs that do not typically require a college degree. While underemployment is not a new phenomenon, it has gained considerable attention since the Great Recession with a growing consensus that underemployed graduates are trapped, unable to escape their low wage jobs.[1] Despite its traction in the media, research which studies underemployment is still in its infancy. We know that recent graduates are nearly ten times more likely to be underemployed than unemployed and that the underemployment rate is countercyclical (Barnichon and Zylberberg, 2019).[2] However, very little is known about the quintessential underemployment duration, whether underemployed graduates are indeed stuck and, if so, what the sources of the underemployment trap are.

This paper studies the features and determinants of underemployment durations by first reporting several new stylized facts. Most prominently, we document negative duration dependence in underemployment. That is, the longer a worker has been underemployed, the less likely they are to transition to an occupation that requires a college degree. We then develop a directed search model which generates duration dependence in underemployment. Finally, we decompose duration dependence into two classic channels: dynamic selection based on unobserved heterogeneity and structural duration dependence generated through the growth and decay of occupation-specific human capital.

We use the National Longitudinal Survey of Youth 1997 (NLSY97) to document three facts. First, the average underemployment duration is nearly eighteen months. Second, the probability an underemployed graduate transitions to a college occupation is decreasing in the length of their underemployment spell. For example, a worker who has been underemployed for a year is nearly 40% less likely to exit underemployment than a newly underemployed worker. Third, an additional underemployment history is associated with 0.13% lower wages in college occupations.

To explain these facts, we develop a model of underemployment grounded in the environments of Gonzalez and Shi (2010) and Menzio and Shi (2011). Workers enter the labor market and direct their search towards non-college or college jobs. The first key ingredient is that workers can be of either limited- or broad-suitability, where a worker's type determines the probability they will produce output at any given college job. As in Gonzalez and Shi (2010), there is symmetric incomplete information regarding a worker's

[1] For a few examples, see "First jobs matter: Avoiding the underemployment trap" by Michelle Weise and "College Grads May Be Stuck in Low-Skill Jobs" by Ben Casselman.

[2] The underemployment rate is typically placed at nearly 40%. See Abel et al. (2014), Barnichon and Zylberberg (2019), BGT and SI (2018), and Jackson (2023).

type and learning occurs through search. Those with a low expected suitability self-select into underemployment and continue to search on the job. It is at this stage where the model's second key ingredient kicks in: underemployment leads the accumulation (decay) of non-college (college) occupation-specific human capital.

The model produces a simple optimality condition relating the marginal cost and benefit of an underemployed worker transitioning to a college job and encompasses the two channels which generate negative duration dependence. First, workers with a longer underemployment duration are more likely to be limited suitability types and are less likely to match with any given college job. This is the unobserved heterogeneity channel. Second, remaining underemployed makes workers more (less) productive in non-college (college) jobs, thereby reducing the marginal benefit of exiting underemployment. This is the human capital dynamics channel.

The model is calibrated to NLSY97 data and used to decompose the model generated duration dependence into the two aforementioned channels. The model can match well, among other moments, the path of transition probabilities between non-college and college jobs and the relationship between underemployment and wages. In our main quantitative exercise, we shut down the human capital dynamics and find that the model with only unobserved heterogeneity can explain 95.27% of the duration dependence observed in the data. When we shut down the unobserved heterogeneity channel, the model fails to generate underemployment. Moreover, the model does not generate nearly enough duration dependence when a worker's type is observable. This underscores the role of unobserved heterogeneity in explaining both the existence of underemployment and the ensuing duration dependence.

Finally, we assess the role of bad luck versus sorting in generating long underemployment durations. We find that even broad-suitable workers who take longer to find their first job, which can occur out of bad luck due to search frictions, do not experience significantly longer underemployment spells than their lucky peers. This again points to the role of sorting in generating duration dependence in underemployment, and implies that there is a weak relationship between a worker's unemployment and underemployment duration. We find that, just as in the model, there is no significant correlation between the length of unemployment and underemployment spells in the data.

Our paper relates to the growing literature which studies underemployment. We are unaware of any study which has documented duration dependence in underemployment or the relation between underemployment and wages in college jobs. Many existing models generate underemployment in random search environments (e.g., Shephard and

Sidibé (2022) and Jackson (2023)).[3] An exception is Barnichon and Zylberberg (2019), where workers direct their search to islands. We develop a competitive search model that generates duration dependence in underemployment, which is absent from the aforementioned models. Finally, we emphasize selection and information frictions as sources of underemployment and the resulting duration dependence.

Underemployment is related to skill mismatch (Guvenen et al., 2020; Lise and Postel-Vinay, 2020; Baley et al., 2022).[4] While these papers have developed innovative approaches for studying the implications of skill mismatch over the business- and life-cycle, we focus on educational mismatch for several reasons. First, the measurement of workers' skills in those papers do not account for skills acquired in college as they are based on test scores measured before most individuals attend college. Second, underemployment has garnered significant attention as many countries implement policies to increase the supply of college graduates. A more thorough understanding of the sources and properties of underemployment has the potential to contribute to such policy discussions.

We draw on the literature which has studied duration dependence in unemployment.[5] Our main findings are consistent with an emerging body of evidence showing that selection can account for a vast majority of duration dependence in unemployment (Mueller et al., 2021; Alvarez et al., 2023; Jarosch and Pilossoph, 2019). Our modelling of skill loss during underemployment is inspired by the literature on skill loss during unemployment (Pissarides, 1992; Ljungqvist and Sargent, 1998). We complement this literature by documenting the relation between underemployment and wages in college occupations and by modelling the loss of occupation-specific human capital during underemployment.

The rest of this paper is organized as follows. Section 2 presents the empirical evidence. Section 3 introduces the model, while Section 4 defines a stationary equilibrium and characterizes the sources of duration dependence. Section 5 presents the quantitative analysis and Section 6 concludes. Appendices are in the online supplementary materials.

2 Empirical Evidence

This section documents three facts: (i) underemployment is more prevalent and persistent than unemployment, (ii) underemployment exhibits negative duration dependence, and (iii) longer underemployment histories are associated with lower wages in college jobs.

[3]See Albrecht and Vroman (2002), Gautier (2002), Dolado et al. (2009), and Coskun (2020).

[4]We measure occupational skill requirements as in Guvenen et al. (2020) and show that they are positively correlated with education requirements. See Figures A.1-A.2.

[5]Recent references include Baydur and Xu (2020), Barnichon and Figura (2015), Doppelt (2016), Fernández-Blanco and Preugschat (2018), and Kospentaris (2021).

Table 1: Frequency and Duration Across Labor Market Statuses

Labor force status	Unemployed	Underemployed	Properly employed
Ratio	0.031	0.392	0.522
Duration (months)	2.39	18.22	22.62

Notes: 5.6% of observations are outside the labor force.

We use the NLSY97 and Occupational Informational Network (O*NET).[6] From the NLSY97, we construct a weekly history of graduates from when they enter the labor market until 2011. An individual's history begins when they graduate with a BA or above and are not enrolled in college thereafter. We arrive at a sample of 996 who obtained a BA or above before 2011 and have a complete set of time-varying individual characteristics.

An individual with a BA or above is underemployed (properly employed) if they work in a non-college (college) occupation. Following Abel et al. (2014) and Jackson (2023), non-college (college) occupations are those where less (more) than 50% of respondents in O*NET releases 5.0-16.0 state that a BA or above is necessary to perform that occupation.[7]

Measuring occupational mobility is prone to measurement error (Moscarini and Thomsson, 2007). While this concern is mitigated in our analysis because we focus on transitions between two broad groups of occupations, we attempt to identify "genuine" switches. Transitions from a non-college to college occupation that are accompanied by a change in employer are treated as genuine. For within-firm switches, we use a three-step correction. First, we measure skill requirements following Guvenen et al. (2020), producing a skill requirement vector, $\mathbf{r}_i$, for each occupation i. Second, we compute the angular distance between two occupations, $\phi(\mathbf{r}_i, \mathbf{r}_j)$, when a worker transitions between occupation i and j. The final step is to label the transition as genuine if $\phi(\mathbf{r}_i, \mathbf{r}_j) \geqslant \bar{\phi}$, i.e., the occupations have sufficiently different skill requirements. The threshold, $\bar{\phi}$, is chosen so that the correlation in skill requirements between occupations is close to zero.[8]

2.1 The Prevalence and Persistence of Underemployment

To highlight the prevalence of underemployment, we calculate the fraction of a respondent's history spent in each labor force status. From the first row of Table 1, respondents

[6] Descriptions of both surveys are in Appendices A.1-A.2.

[7] Table A5 lists occupations around the 50% threshold while the ten most common college and non-college occupations are listed in Table A6. The 50% cutoff produces an underemployment rate that is similar to alternative measures of education requirements (BGT and SI, 2018; Barnichon and Zylberberg, 2019). Appendix B assesses the robustness of our findings to alternative measures of educational requirements.

[8] Appendices A.3-A.4 provide more details on this correction.

spent 39.2% of their post-graduate career underemployed. The second row of Table 1 shows that the average underemployment duration is around 18 months.

2.2 Duration Dependence

We define exiting underemployment to be when a worker transitions from a non-college to a college occupation between week $t-1$ and t.[9] Our objective is to estimate the negative exponential relationship between the probability of transitioning from underemployment to proper employment and the worker's underemployment duration.[10] Specifically, we estimate the following via weighted nonlinear least squares:

$$D(\tau) = b_1 + (1-b_1)\exp(-b_2 \times \tau), \tag{1}$$

where $D(\tau)$ is the average exit probability at duration τ relative to the average exit probability of workers who have been underemployed for less than one month. To estimate (1), we need estimates of the average exit probabilities at each duration τ. These are obtained by estimating

$$y_{it} = \beta\tau_{it} + \Gamma \cdot X_{it} + \delta_t + \epsilon_{it}, \tag{2}$$

where y_{it} is an indicator for whether individual i exited underemployment during period t, τ is underemployment duration, and δ_t contains month and year fixed effects. The vector, X, contains gender, race, age, gender interacted with race, ASVAB quartile, family income, outstanding student loan debt, highest degree, gender interacted with highest degree, undergraduate GPA, undergraduate major (STEM or Arts and Social Sciences), and job satisfaction.[11] We then compute the predicted transition probabilities at each duration $\tau \in \{1, \ldots, 24\}$ relative to $\tau = 0$.

Figure 1 displays the results. The triangles and circles represent the predicted transition probabilities generated by equation (2) with and without individual level controls, respectively. The curves are the result of estimating (1) on each set of relative transition probabilities. Controlling for observable characteristics considerably attenuates the duration dependence profile. While we find that demographics and ASVAB score have a sizeable effect on the duration dependence profile, the worker's job satisfaction has the

[9] The duration dependence is similar if we allow for three weeks between transitions. See Figure A.5(a).

[10] This approach follows Jarosch and Pilossoph (2019). Workers with an underemployment duration $\geqslant 24$ months are grouped together, as there are rarely transitions from underemployment at such durations. Figure A.5(b) shows that the results are largely unchanged under a maximum duration of 30 months.

[11] Figure A.4 shows that job satisfaction is significantly higher in college occupations. Table A7 shows that STEM (Arts and Social Sciences) majors spent 32.5% (42.9%) of their labor market history underemployed.

largest impact on attenuating the profile.[12] After controlling for observable characteristics, we still observe a decline in the relative transition probability over the first year of underemployment, before leveling off at higher underemployment durations.

The shape of the decline in the relative exit probability is indicative of selection. If workers, based on unobservable characteristics, have different propensities to exit underemployment, then workers with a high exit probability will quickly leave while the long term underemployed will be primarily comprised of those with a low exit probability. There could also be structural forces causing each individual's probability of exiting underemployment to decline. To examine if there is evidence supporting this, we proceed to study the relationship between underemployment and wages.

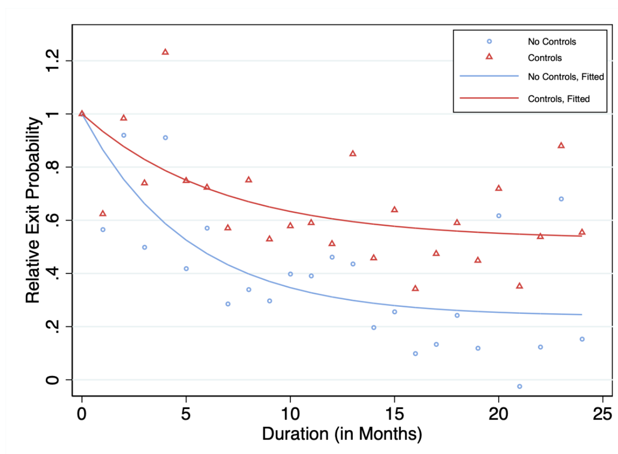


Figure 1: Duration Dependence in Underemployment

2.3 Wages and Underemployment

Longer unemployment durations are associated with lower wages (Ortego-Marti, 2016; Laureys, 2021). This fact is consistent with two prominent explanations for structural duration dependence: human capital depreciation and statistical discrimination. In the former, a worker's skills depreciate over their unemployment spell, which lowers their productivity and chance to find a job. In the latter, a longer unemployment duration signals that the worker is less productive. To assess whether workers with a longer underemployment history earn lower wages in college jobs, we estimate

$$w_{it} = \alpha \text{Underhis}_{it} + \beta \text{College}_{it} + \mu \text{Underhis}_{it} \times \text{College}_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \quad (3)$$

[12]See Figure A.3 for a detailed analysis on the attenuation of the duration dependence profile.

Table 2: Unemployment, Underemployment, and Wages

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0145***		-0.0145***	-0.0139***		-0.0136***
	(0.0009)		(0.0009)	(0.0011)		(0.0011)
Underhis		0.0003***	0.0003***		0.0007***	0.0006***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
Unhis × College				-0.0009		-0.0004
				(0.0013)		(0.0013)
Underhis × College					-0.0020***	-0.0019***
					(0.0002)	(0.0002)
Occupation (2-digit) FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.791	0.790	0.791	0.782	0.782	0.783

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

where w_{it} is individual i's log wage in period t, Underhis is accumulated experience in non-college occupations, and College is a dummy for being employed in a college occupation. The vector X includes a cubic in potential experience, annual regional and national unemployment rate, a quadratic in age, family income, student loan debt, job satisfaction, region, and two-digit industry fixed effects. Finally, δ_i is an individual fixed effect.

Table 2 presents the results. Column (6) is our preferred specification and shows that an additional month of underemployment is associated with 0.06% higher wages in non-college jobs and 0.13% lower wages in college occupations.[13] Moreover, we find that an additional month of unemployment (Unhis) is associated with a 1.36-1.45% decline in wages, which is consistent with prior literature (Addison and Portugal, 1989; Neal, 1995).

2.4 From Empirics to Theory

Before transitioning to the theory, this section gives a preview of how the model will generate duration dependence in underemployment and the results of Table 2. As mentioned above, the patterns in Figure 1 are indicative of selection. To allow for this, we consider workers who are heterogeneous in their (unobservable) suitability for college jobs. Workers with a high (low) suitability for college jobs will quickly (slowly) exit un-

[13] We conduct several robustness exercises. First, we use one-digit industry and occupation fixed effects (Table A8). Second, we control for one-digit or two-digit occupation fixed effects in all specifications to address the concern of occupation heterogeneity (Altonji and Shakotko, 1987) (Tables A9 and A10). Finally, we include month and year fixed effects (Table A11). The results are similar across all specifications.

deremployment. Thus, the exit rate will decrease over the underemployment spell as the composition of workers shifts towards those with limited suitability.

To reconcile the findings in Table 2, we assume underemployed workers accumulate non-college occupation-specific human capital, making them more productive in non-college jobs. At the same time, workers do not utilize their college-specific skills while underemployed. Therefore, a worker who transitions to a college job after a long underemployment spell will be less productive than one who quickly left underemployment. The former effect leads to higher wages in non-college jobs, while the latter reduces wages in college jobs. Occupation-specific human capital dynamics also generate duration dependence, as the benefit of exiting underemployment decreases as workers become more (less) productive in non-college (college) jobs.

Our modelling of human capital aligns with evidence that human capital is occupation-specific (Kambourov and Manovskii, 2009). To further support this approach, we study how the association between underemployment and wages varies with the distance in skill requirements between college and non-college jobs. Suppose that a worker transitions from non-college occupation i to college occupation j. If the distance $\phi(\mathbf{r}_i, \mathbf{r}_j)$ is larger, then skills required by j would not have been used as intensively while employed at i, causing skills used by j to decay at a greater rate and larger wage losses. Table A4 shows that this is the case in the data, as the association between underemployment and college wages becomes even more negative as the distance in required skills increases.

3 Model

Time is discrete and goes on forever. There is a measure one of workers and a large measure of firms. All agents are risk neutral and share the discount factor $\beta \in (0, 1)$. Firms are indexed by $\chi \in X = \{n, c\}$, where n (c) denotes a non-college (college) job.

Workers are ex-ante heterogeneous in their suitability type $i \in \{L, H\}$ for college jobs, where the mass of type H workers is $\pi \in (0, 1)$. We refer to type H (L) workers as broad- (limited-) suitable. A type i worker is suitable for any given college job with probability a^i, where $a^H > a^L$. Workers produce zero output in college jobs they are unsuitable for. There is symmetric incomplete information regarding a worker's suitability type.

Workers are also heterogeneous in their labor market history, where $\upsilon \in \Upsilon = \{0, 1, \ldots, \bar{\upsilon}\}$ is the number of periods a worker has been unemployed and $\tau \in T = \{0, 1, \ldots, \bar{\tau}\}$ is the worker's underemployment history (experience in non-college jobs). A worker's history, (υ, τ), is public information.

Upon meeting a worker, firms with college jobs observe a private signal which perfectly identifies unsuitable workers, allowing them to hire suitable workers and reject unsuitable ones. Neither workers nor other firms observe the signal. Additionally, firms operate a technology that maps one unit of suitable labor into $y_\chi(\tau)$ units of output where $y_\chi\colon T \to \mathbb{R}_+$, y_n (y_c) is weakly increasing (decreasing) in τ, and $y_c(\tau) > y_n(\tau)$ for all $\tau \in T$.

The labor market is organized in a continuum of submarkets indexed by $\omega = (\chi, v, \tau, x)$. In submarket ω, type χ firms search for workers with history (v, τ) and offer suitable workers a contract worth $x \in \mathbb{R}$ in lifetime utility.

Each period is divided into four stages: search, matching, entry/production, and exit. In stage 1, firms incur a cost k_χ to post a type χ vacancy and workers select a submarket to search in. Employed workers are endowed with $\lambda \in [0, 1]$ units of search intensity.

In stage 2, suitable workers and vacancies search for each other. Let $v(\omega)$ denote the measure of vacancies in submarket ω. Further, $u^i(\omega)$ and $e^i(\omega)$ denote the measure of unemployed and employed workers, respectively, of suitability type i searching in submarket ω. The effective measure of suitable workers is $\psi(\omega) = \sum_i a^i(\omega)[u^i(\omega) + \lambda e^i(\omega)]$, where $a^i(\omega) = 1$ (a^i) if $\chi = n$ (c). The number of matches is given by the constant returns to scale matching technology $F(\psi(\omega), v(\omega))$. Defining $\theta(\omega) \equiv v(\omega)/\psi(\omega)$ as tightness, suitable unemployed workers match with probability $p(\theta) = F/\psi$, where $p(\theta)$ is strictly increasing and concave, $p(0) = 0$, and $p(\infty) = 1$. Suitable workers searching on the job find a match with probability $\lambda p(\theta)$. Vacancies are filled with probability $q(\theta(\omega)) = F/v$, where $q(\theta)$ is strictly decreasing and convex, $q(0) = 1$, and $q(\infty) = 0$.

In stage 3, a measure δ of workers enter the labor market unemployed. Matches (unemployed workers) produce $y_\chi(\tau)$ (b) units of output. After producing at a college job, workers with $\tau > 0$ regain their skills with probability ϕ. Workers exit the market with probability δ in stage 4. There are no transitions from employment to unemployment.

Let μ denote the worker's expectation that they will produce output in a college job. Their initial belief is $\mu_0 = \pi a^H + (1 - \pi) a^L$. Unemployed workers who search for and do not find a college job update their beliefs, using Bayes rule, to

$$\hat{\mu} \equiv H(p, \mu) = a^H - \frac{(a^H - \mu)(1 - p a^L)}{1 - p\mu}, \tag{4}$$

where $p = p(\theta)$. Underemployed workers update their beliefs to $H(\lambda p, \mu)$. Equation (4) also captures firms' beliefs about a worker's suitability, as there is symmetric incomplete information. Moreover, (v, τ) is a sufficient statistic for μ as (v, τ) captures how many times the worker's expected suitability was updated according to (4).[14]

[14]We show in Proposition 1 that employment in college jobs is an absorbing state. Therefore, a worker with history (v, τ) was unemployed for v periods before becoming underemployed for τ periods.

The contract space is complete, which ensures that contracts offered by firms are bilaterally efficient (Menzio and Shi, 2009, 2011). As multiple contracts can deliver bilateral efficiency, and wage data is used to estimate the model, we are explicit about the employment contract. We follow Schaal (2017) and Baley et al. (2022) by assuming firms offer a contract where wages equal match output and workers incur a one-time hiring fee to compensate firms for their recruitment costs.

4 Equilibrium

4.1 Value Functions, Free Entry, and Equilibrium Definition

As employment contracts are bilaterally efficient, it is without loss of generality to solve the model where submarkets are indexed by the value delivered to the worker, x, and by characterizing the joint surplus of a match. Let $V_{u,\chi}(v,\mu)$ denote the lifetime utility of an unemployed worker, measured at the beginning of stage 3, with unemployment history v, expected suitability μ, and searches in a submarket with type χ jobs. They produce b and remain in the labor market with probability $1-\delta$. Suppose, in the next period, that the worker searches for a non-college job in submarket $\omega = (n, \hat{v}, 0, x)$ where $\hat{v} = \min\{v+1, \bar{v}\}$. The worker finds a match and earns the continuation value, x, with probability $p(\theta(\omega))$. If the worker does not find a match, their continuation value is $V_u(\hat{v},\mu) = \max\{V_{u,n}(\hat{v},\mu), V_{u,c}(\hat{v},\mu)\}$. It follows that $V_{u,n}(v,\mu)$ satisfies

$$V_{u,n}(v,\mu) = b + \beta(1-\delta)\{V_u(\hat{v},\mu) + \max_x p(\theta(n,\hat{v},0,x))(x - V_u(\hat{v},\mu))\}. \tag{5}$$

Now suppose that an unemployed worker searches for a college job. Workers expect to find a job with probability $\mu p(\theta(\omega))$. Those who do not find one update their beliefs to $\hat{\mu} = H(p(\theta(\omega)),\mu)$. The value of searching for a college job satisfies:

$$V_{u,c}(v,\mu) = b + \beta(1-\delta)\{V_u(\hat{v},\hat{\mu}) + \mu \max_x p(\theta(c,\hat{v},0,x))(x - V_u(\hat{v},\hat{\mu}))\}. \tag{6}$$

Let $V_{e,n}(v,\tau,\mu)$ denote the joint value of a match between a non-college job and a worker with characteristics (v,τ,μ). The match produces $y_n(\tau)$ units of output. If the worker remains in the market, they transition to another job with probability $\lambda\mu p(\theta(\omega))$ and the worker's (firm's) continuation value is x (0). If the match survives, the continuation value is $V_{e,n}(v,\hat{\tau},\hat{\mu})$, where $\hat{\tau} = \min\{\tau+1,\bar{\tau}\}$. Thus, $V_{e,n}(v,\tau,\mu)$ satisfies

$$V_{e,n}(v,\tau,\mu) = y_n(\tau) + \beta(1-\delta)\{V_{e,n}(v,\hat{\tau},\hat{\mu}) + \lambda\mu \max_x p(\theta(c,v,\hat{\tau},x))(x - V_{e,n}(v,\hat{\tau},\hat{\mu}))\}. \tag{7}$$

Equation (7) implies that underemployed workers only search for college jobs. As shown in Proposition 1 in the supplementary materials, this is because workers employed in type χ jobs cannot generate additional surplus by transitioning to another type χ job.

For a worker with characteristics (v, τ, μ) employed at a college job, the match produces $y_c(\tau)$ units of output. With probability ϕ, the worker regains their skills and produces $y_c(0)$ for the remainder of the match. The joint value, $V_{e,c}(v, \tau, \mu)$, satisfies

$$V_{e,c}(v, \tau, \mu) = y_c(\tau) + \beta(1-\delta)\{\phi V_{e,c}(v, 0, \mu) + (1-\phi)V_{e,c}(v, \tau, \mu)\}. \tag{8}$$

Proposition 1 also shows that proper employment is an absorbing state, as properly employed workers cannot generate additional surplus by moving to a non-college job.

The firm's cost to create a vacancy is k_χ whereas the benefit from posting a vacancy in submarket $\omega = (\chi, v, \tau, x)$ is $q(\theta(\chi, v, \tau, x))\{V_{e,\chi}(v, \tau) - x\}$.[15] In submarkets visited by workers, tightness is consistent with firms' incentives to create vacancies if and only if

$$k_\chi \geqslant q(\theta(\chi, v, \tau, x))\{V_{e,\chi}(v, \tau) - x\}, \tag{9}$$

and $\theta(\chi, v, \tau, x) \geqslant 0$ with complementary slackness.

Definition 1. A stationary recursive equilibrium consists of a belief function $\hat{\mu}(p, \mu)$, tightness function $\theta(\omega)$, value and policy function for unemployed workers, $V_u(v, \mu)$ and $\omega_u^*(v, \mu)$, joint value and policy function, $V_{e,\chi}(v, \tau, \mu)$ and $\omega_{e,\chi}^*(v, \tau, \mu)$, and a distribution of workers that satisfies the following conditions. First, $\hat{\mu}(p, \mu)$ is given by (4). Second, $\theta(\omega)$ satisfies (9) and the slackness condition for all ω. Third, $V_u(v, \mu) = \max\{V_{u,n}(v, \mu), V_{u,c}(v, \mu)\}$ where $V_{u,n}(v, \mu)$ and $V_{u,c}(v, \mu)$ satisfy (5)-(6) and $\omega_u^*(v, \mu)$ is the associated policy function. Fourth, $V_{e,\chi}(v, \tau, \mu)$ for $\chi \in X$ satisfies (7)-(8) and $\omega_{e,\chi}^*(v, \tau, \mu)$ are the associated policy functions. The distribution of workers satisfies the laws of motion specified in Appendix D.1.

As established by Menzio and Shi (2010, 2011) for directed search models with free entry and bilateral efficiency and Schaal (2017) for similar environments with two-sided heterogeneity, a recursive equilibrium exists and is block-recursive (BRE). As workers self-select into submarkets based on their observable characteristics, firms know they will only meet one type of worker in their respective submarket. Additionally, the hiring protocol and matching function specification of Gonzalez and Shi (2010) implies that firms do not need to keep track of the composition of suitable workers in each submarket.

[15] $V_{e,\chi}(v, \tau)$ and $V_{e,\chi}(v, \tau, \mu)$ are equivalent as (v, τ) is a sufficient statistic for μ.

Hence, tightness in each submarket is independent of the distribution of workers across employment statuses and the composition of worker suitability.

4.2 Duration Dependence in Underemployment

This subsection describes how the model generates duration dependence in underemployment and distinguishes between the underlying channels.[16] To begin, we combine the entry condition (9) with (7), which allows us to write the problem of an underemployed worker as a choice of tightness. The first order condition is given by

$$k_c \geqslant p'(\theta^*_{c,v,\tau})(V_{e,c}(v,\tau,\mu) - V_{e,n}(v,\tau,\hat{\mu})), \tag{10}$$

and $\theta^*_{c,v,\tau} \geqslant 0$ with complementary slackness. As workers exit underemployment with probability $\mu\lambda p(\theta^*_{c,v,\tau})$, there are two channels which generate duration dependence in underemployment. First, workers with higher τ have a lower expected suitability, μ. This is the selection channel. Second, underemployment reduces (increases) the worker's productivity in college (non-college) jobs, putting downward pressure on the surplus generated by escaping underemployment, tightness $\theta^*_{c,v,\tau}$, and the matching probability $p(\theta^*_{c,v,\tau})$. This is the human capital dynamics channel.

How can the model distinguish between selection and human capital dynamics? Recall that workers earn a wage that is equal to match output. Moreover, match output, $y_\chi(\tau)$, is independent of a worker's suitability type. This is our identifying restriction, as it means that the model's relationships between underemployment history and wages are completely driven by the growth and decay of occupation-specific human capital.

There is an additional outcome that can distinguish between the two channels. Consider the path of transition probabilities from underemployment to proper employment $\{\mu_\tau \lambda p(\theta^*_{c,v,\tau})\}_{\tau=1}^{\bar{\tau}}$. Proposition 2 in the supplementary materials shows that the model with only human capital dynamics generates a transition path that is generally decreasing and concave, especially at higher values of τ. The intuition is that as τ increases, the surplus generated by a worker exiting underemployment decreases. As the surplus continues to decline, each additional period of underemployment causes a larger relative change in the match surplus, meaning that the responses of vacancies, tightness, and the exit probability become progressively larger as the worker remains underemployed.

With unobserved heterogeneity only, the path is generally decreasing and convex. This is because the path's shape is primarily determined by the evolution of the expected suitability, μ_τ, which typically has a convex shape (see Proposition 3 in the supplemen-

[16]We thank an anonymous referee whose suggestions led to the development of this subsection.

tary materials).[17] Intuitively, workers learn more from not finding a college job early in their underemployment spell, causing them to rapidly downgrade their beliefs. As the worker remains underemployed, the changes in μ_τ become progressively smaller, leading to a transition path that becomes flat as τ increases.

5 Quantitative Analysis

5.1 Calibration

A unit of time is one month, $\bar{v} = 12$, and $\bar{\tau} = 24$. The matching technology is $F(\psi, v) = \frac{\psi v}{\psi + v}$ and the production functions are $y_\chi(v, \tau) = g_\chi e^{(d_{\chi,v}(v-1)+d_{\chi,\tau}\tau)}$ for $\chi \in X$, which allows for skill loss during unemployment. There are 16 parameters. The discount factor is $\beta = 0.95^{1/12}$ and we normalize the economy by setting $g_c = 1$. The remaining 14 parameters are calibrated via method of simulated moments (MSM) to match 33 moments. The first moment is $b/[\text{Average labor productivity}] = 0.71$ (Hall and Milgrom, 2008). Second is the college job wage premium, i.e., how much higher a worker's wage is in a college job than an observationally equivalent worker's in a non-college job. Following the approach of Barnichon and Zylberberg (2019) gives a premium of 25.97%.[18] We also target the regression coefficients in column (6) of Table 2. The remaining moments are an unemployment rate of 8.1%,[19] underemployment rate of 41.6%, average number of months spent unemployed before entering underemployment for the first time (2.147), and the path of 24 transition probabilities from non-college to college jobs that controls for observable characteristics (the red line in Figure 1).

The skill parameters $\{d_{\chi,v}, d_{\chi,\tau}\}$ for $\chi \in X$ are chosen to match the regression coefficients in column (6) of Table 2. While each of the 10 remaining parameters impacts all of the remaining moments, one can view b as targeting the $b/[\text{labor productivity}]$ ratio, g_n the college job wage premium, and $\{k_n, k_c\}$ the unemployment and underemployment rates as the entry costs affect both the amount and composition of vacancies.

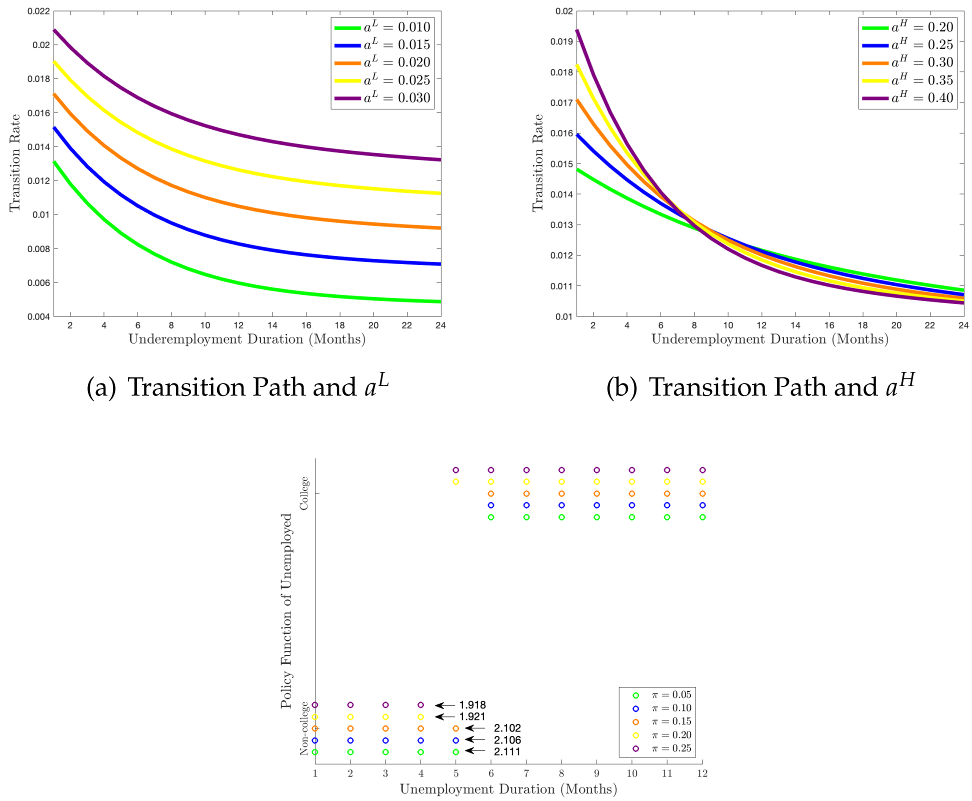
The unobserved heterogeneity parameters, $\{\pi, a^L, a^H\}$, are chosen to match two features of the path of transition probabilities and the average number of months spent unemployed before becoming underemployed for the first time. The lower bound of the transition path is informative of a^L. The reason is that $\mu \to a^L$ as $\tau \to \bar{\tau}$. In words, workers who remain underemployed learn that they are likely a limited-suitable worker.

[17] The matching probability, $p(\theta)$, does not vary much with τ in our quantitative analysis as there are offsetting effects of an increase in τ on the surplus generated by exiting underemployment.

[18] See Appendix C.1 for more details.

[19] We target the nonemployment rate because, in the data, there are a significant number of transitions from not in the labor force to employment.

It follows that the transition path at higher underemployment durations is largely determined by a^L as $\lambda\mu_\tau p(\theta_\tau) \to \lambda a^L p(\theta_\tau)$ as $\tau \to \bar{\tau}$. Figure 2(a) shows that the transition path levels off at a higher (lower) transition probability with a higher (lower) value of a^L.



(a) Transition Path and a^L

(b) Transition Path and a^H

(c) U2N Duration and π

Figure 2: Identification of $\{\pi, a^L, a^H\}$

The "convexity" of the transition path is informative for a^H. This is because, as a^H increases, broad-suitable workers leave underemployment at a higher rate. So, workers who remain underemployed quickly realize they are likely a limited-suitable worker. Figure 2(b) shows that as a^H increases, the transition path declines at a higher rate initially, as workers quickly downgrade their expected suitability, before leveling off and giving a transition path with a more pronounced convex shape.

We then utilize the occupation choice of an unemployed worker to calibrate π. Recall that a worker's initial expected suitability is $\mu_0 = \pi a^H + (1-\pi)a^L$. Initial beliefs are important in determining which type of job an unemployed worker searches for. If the worker has lower initial beliefs, they will spend more time searching for a non-college job as there is little expected benefit to searching for a college job. With a^L and a^H pinned down by the lower bound and convexity of the transition path, π is the only free parameter in determining the initial beliefs. We calibrate π by targeting the average number of months between a worker entering the labor market and beginning their first underemployment spell, which we refer to as the U2N duration. Figure 2(c) displays the type of job workers search for at each unemployment duration. For example, at $\pi = 0.05$, the worker searches for a non-college job for their first five periods of unemployment, and then uses the remainder of their unemployment spell to search for a college job. The U2N duration associated with each value of π is listed next to the black arrows. As π decreases, workers search longer for a non-college job, causing the U2N duration to increase.

There are three remaining parameters $\{\lambda, \delta, \phi\}$ that can be interpreted as being chosen to fine-tune the model's fit to the 33 moments. Intuitively, λ directly affects the job-to-job transition probability of suitable workers, $\lambda p(\theta)$, while δ impacts the expected duration of a match and therefore tightness, θ. Therefore, adjusting $\{\lambda, \delta\}$ improves the model's fit to the path of transition probabilities. As seen in Figure 2(c), changing π can cause discrete jumps in the U2N duration when a worker changes the number of months spent searching for a non-college job. Changing ϕ impacts tightness and therefore the average unemployment duration, helping better the model's fit to the U2N duration.

We now introduce the estimation procedure. Denoting $\tilde{m}$ (m) as the vector of 33 model generated (empirical) moments, the vector of 14 parameters, $\hat{\vartheta}$, is given by

$$\hat{\vartheta} = \arg\min \left(\tilde{m} - m\right)' W \left(\tilde{m} - m\right). \tag{11}$$

We use two weighting matrices, W. The first, and one we use throughout this section, is $W = I/m^2$, where I is the identity matrix. This scaled identity matrix minimizes the sum of squared percentage deviations between the model and empirical moments and does not place more weight on moments which are larger in magnitude. Table 3 and Figure 3 show that the model matches the data well. Table 4 displays the parameter values. We also use the inverse variance-covariance matrix of the empirical moments for W, which produces similar parameter values. See Appendix C.2 for more details.

5.2 Decomposing Duration Dependence

This section evaluates the relative contributions of unobserved heterogeneity and human capital dynamics in generating duration dependence. Beginning with Figure 4(a), we present the transition path from the data, model, and model without skill dynamics during underemployment by setting $d_{n,\tau} = d_{c,\tau} = 0$. The model with only unobserved heterogeneity generates a substantial amount of duration dependence.

We next ask what percentage of the decline in the transition probability at each under-

Table 3: Model and Data Comparison

Moment	Target	Model	Moment	Target	Model
Unemployment rate	0.081	0.081	$\partial \log(w_n)/\partial v$	-0.014	-0.014
Underemployment rate	0.416	0.414	$\partial \log(w_c)/\partial v$	-0.014	-0.014
U2N duration	2.147	2.111	$\partial \log(w_n)/\partial \tau$	0.001	0.001
College job premium	0.260	0.259	$\partial \log(w_c)/\partial \tau$	-0.001	-0.001
b/[Average labor productivity]	0.710	0.707	-	-	-

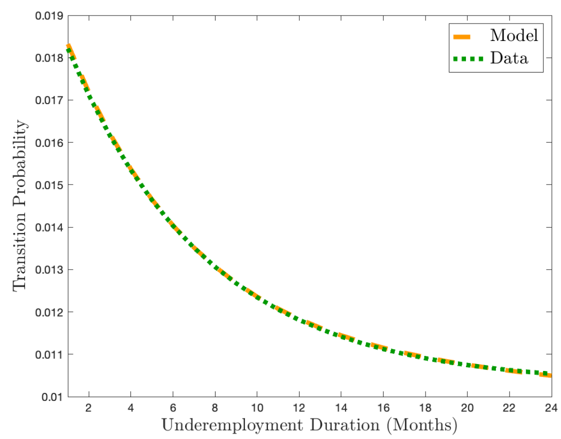


Figure 3: Duration Dependence in the Model and Data

employment duration relative to the transition probability at $\tau = 1$ observed in the data can be explained the model with unobserved heterogeneity only. Figure 4(b) illustrates that the model with only unobserved heterogeneity explains at least 94% of the decline.

To arrive at an aggregate decomposition, we compute the weighted average of the fraction explained by unobserved heterogeneity over all τ. The weights are the fraction of underemployed workers who, in the steady-state, are employed at each τ. After removing human capital dynamics during underemployment, the model can explain 95.27% of the duration dependence. Therefore, unobserved heterogeneity is by far the biggest driver of duration dependence in our model.[20]

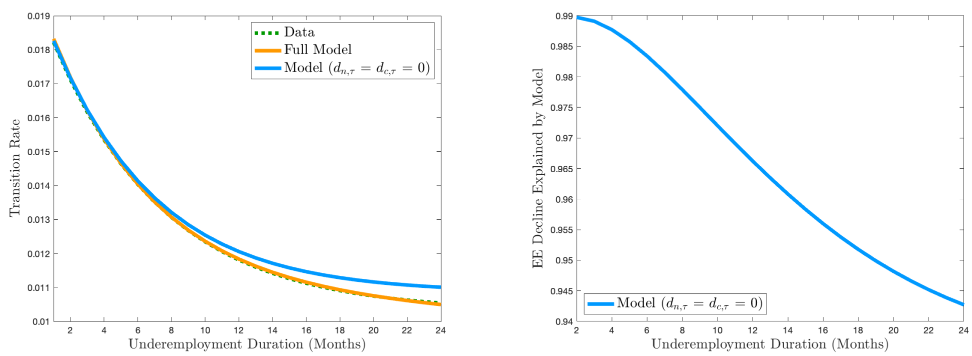
Next, we turn off the unobserved heterogeneity channel by setting $a^L = a^H = 1$ while all other parameters take the values in Table 4. Our main finding here is that workers never select into underemployment. Thus, both the duration dependence and existence of underemployment are closely tied to the presence of unobserved heterogeneity.

To conclude this section, we ask to what extent information frictions matter in gener-

[20] Appendix A.11 provides suggestive evidence for the presence of unobserved heterogeneity in the data.

Table 4: Parameter Values

	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.354
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.049
g_n	Non-college productivity	0.745	ϕ	Pr. of regaining college skills	0.006
b	Utility while unemployed	0.611	$d_{c,v}$	College skill loss: unemp.	-0.014
k_n	Non-college vacancy cost	2.167	$d_{c,\tau}$	College skill loss: underemp.	-0.001
k_c	College vacancy cost	2.054	$d_{n,v}$	Non-college skill loss: unemp.	-0.014
λ	Employed search intensity	0.851	$d_{n,\tau}$	Growth of non-college skills	0.001

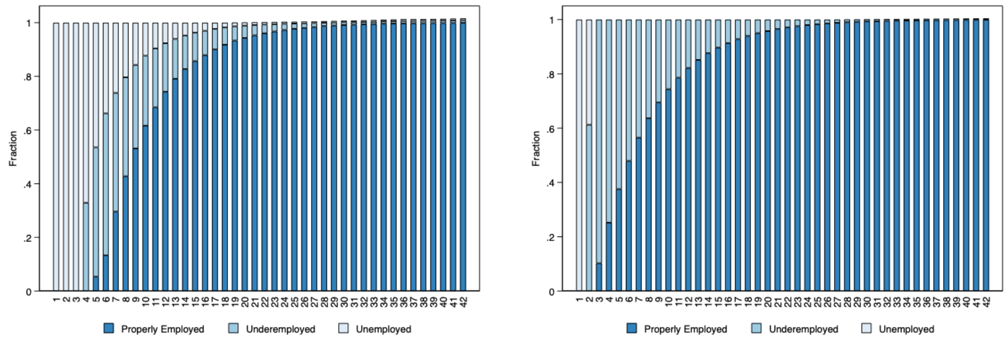


(a) Path of Transition Probabilities

(b) Fraction Explained at each τ

Figure 4: Duration Dependence Decomposition

ating underemployment and duration dependence. To answer this, we solve the version of our model where a worker's suitability type is public information (see Appendix E for details). Two findings emerge from removing information frictions. First, under the calibrated values in Table 4, broad-suitable workers never search for a non-college job. Therefore, the pool of underemployed workers contains only limited-suitability workers. Second, the full information model generates a negligible amount of duration dependence, as the transition probability decreases from 0.011 at $\tau = 1$ to 0.010 at $\tau = 24$. It follows that information frictions are a key ingredient in generating an amount of duration dependence commensurate with what is observed in the data.



(a) Unlucky Workers

(b) Lucky Workers

Figure 5: Percentage of Time Spent in Various Labor Market Statuses

5.3 Sorting and Bad Luck

To this point, our main quantitative finding is that a vast majority of the duration dependence observed in the data is accounted for by unobserved heterogeneity. A related and open question is what role bad luck plays in generating long underemployment durations. To evaluate the role of bad luck, we simulate the model and compare two groups of broad-suitable workers. The first group, "lucky", are those who find their first job within three months of entering the labor market. The second, "unlucky", are those who take more than three months to find their first job. Figure 5 compares the fraction of each month spent in each labor market status across the two groups. From Figure 5(a), unlucky workers gradually transition into underemployment after spending their first three months unemployed. Despite a slow start, the average underemployment duration among the unlucky (lucky) group is 5.58 (5.54) months, which suggests that there is little relation between the length of an unemployment and underemployment spell.

Table 5 shows that, similar to the model, there is a weak correlation between unemployment and underemployment duration in the data.[21] This is consistent with the intuition above. Workers exit underemployment relatively quickly (slowly) if they are a broad-suitable (limited-suitable) worker, irrespective of their unemployment history.

To further understand the model generated correlation between a worker's unemployment and underemployment duration, recall that the identifying restriction pins down the skill growth and decay parameters. Moreover, the unobserved heterogeneity parameters are chosen to match the shape of the transition path and U2N duration. Through this identification strategy, we found $a^H = 0.354$ and $a^L = 0.023$, meaning that broad-suitable workers exit underemployment at a much higher rate than limited-suitable work-

[21] Appendix A.9 contains more details on the construction of Table 5.

Table 5: Correlation between Unemployment and Underemployment Durations

	Data		Model
	Unconditional	Conditional	
First U2N transition	-0.008 (0.854)	0.020 (0.898)	-0.025
All U2N transitions	-0.024 (0.446)	0.035 (0.742)	

Notes: P-values in parentheses.

ers. Moreover, the skill loss parameters are not large enough to generate lasting effects on the productivity of the worker. Hence, a long unemployment spell does not have persistent effects on an unlucky worker's chances to exit underemployment.

5.4 Discussion of Main Assumptions

To support the notion that college graduates are uncertain about their type and learn over time, we use the National Longitudinal Survey of Youth 1979 (NLSY79), where respondents indicated their expected occupation in 5 years and at age 35, allowing us to construct forecast errors between their actual and expected occupations. We report three main findings here and delegate the details to Appendix A.8. First, most college graduates make forecast errors regarding their future occupation. Second, those who make larger forecast errors are more likely to become underemployed, especially early in their career. The third finding is related to the correlation between graduates' actual and predicted forecast errors. Under the null hypothesis that workers know their type, the correlation between the actual and predicted errors is zero (Chahrour and Ulbricht, 2023). We find a statistically significant positive correlation. Moreover, the positive correlation suggests that graduates learn their type over time (Baley et al., 2022).

Our identifying restriction is that output is independent of a worker's suitability type. Under this assumption, the relationship between wages and a worker's employment history is completely driven by human capital dynamics. Moreover, the identifying restriction acts to dampen the role of information frictions in generating duration dependence, as a worker's expected suitability does not directly impact the surplus of matching with a college job. Suppose instead that the production function in college jobs was given by $y_c^i(\tau)$ with $y_c^L(\tau) < y_c^H(\tau)$ for all $\tau \in T$. Under the employment contracts used throughout this paper, the average wage of workers with history $\hat{\tau}$ relative to those with τ would be

$$\frac{w_c(\hat{\tau})}{w_c(\tau)} = \frac{y_c^L(\hat{\tau}) + \mu_{\hat{\tau}}(y_c^H(\hat{\tau}) - y_c^L(\hat{\tau}))}{y_c^L(\tau) + \mu_{\tau}(y_c^H(\tau) - y_c^L(\tau))}. \tag{12}$$

From (12), a key determinant of the relative wages is the rate at which beliefs evolve, $\mu_{\hat{\tau}}/\mu_{\tau}$. As shown in Proposition 3, if μ is low enough upon entering underemployment, the beliefs follow a convex shape with a steep initial decline and then level off at higher values of τ. We find little evidence in the data to support such a convex pattern in the association between underemployment and wages in college jobs (see Appendix A.10). Moreover, we find a relative decline that is approximately linear, which is the pattern generated by our quantitative model.

A second determinant of the relative wage decline in (12) is the difference in output across suitability types, $y_c^H(\tau) - y_c^L(\tau)$. Even if a worker's suitability type affects match output, the data suggest that these output differences are not quantitatively significant. Figure A.7 shows that the decline in relative wages is minimal relative to that in the exit probability, suggesting that differences in output across suitability types are small. To further solidify this point, we calibrate a version of the model where $y_c^H(\tau) = \alpha y_c^L(\tau)$. The calibration strategy is unchanged except that instead of only targeting the regression coefficients from column (6) of Table 2, we target the path of the relative wages in college jobs shown in Figure A.7(a), as the shape of the relative decline in wages is informative about the differences in productivity across suitability types. Through this calibration, we find $\alpha = 1.07$. Thus, the calibration assigns modest differences in productivity across suitability types. Moreover, we still find that unobserved heterogeneity explains the majority of duration dependence in underemployment (98.80%). See Appendix C.3 for the specifics on this version of the model and its calibration.

The ability to search on the job is an important avenue through which broad-suitable workers exit underemployment, and makes workers more willing to enter underemployment in light of the uncertainty they face about their suitability. Moreover, the search intensity of underemployed workers, λ, is influential in determining the amount of information underemployed workers acquire from not finding a college job.[22] The inclusion of on the job search is motivated by the fact that 79.5% of transitions from underemployment to proper employment in our NLSY97 sample occur through a job-to-job transition.

Finally, the model does not feature interactions between those with and without a college degree. While such interactions are important for the cyclicality of underemployment (Barnichon and Zylberberg, 2019), we abstract from including less-educated workers because the block-recursive nature of the equilibrium would result in workers self-selecting into different submarkets based on their educational attainment (and other

[22]Recall that underemployed workers update their beliefs according to $H(\lambda p, \mu)$ in (4). All else being equal, a higher λ implies a higher job finding probability of suitable workers, meaning workers will place more weight on the possibility they are a limited-suitable type in the event they do not find a college job.

observable characteristics). Therefore, the entry of firms and duration dependence in underemployment would not be impacted by having less-educated workers in the model.

One way to generate interactions between less- and highly educated workers is to depart from the directed search environments of Gonzalez and Shi (2010) and Menzio and Shi (2011). One could also assume output from non-college and college jobs are imperfect substitutes in the production of a final good. While these are interesting extensions, they would bring substantial complications, as agents would need to keep track of the distribution of workers.

5.5 Robustness of Duration Dependence Decomposition

To assess the sensitivity of our main quantitative result, the decomposition of duration dependence, we conduct several robustness exercises.

Recall that we define a college job as an occupation where at least 50% of O*NET respondents indicate that a bachelors degree or above is required to perform that job. We consider two alternative definitions. First, we adjust the threshold from 50% to 42.27% as this threshold corresponds to where 60% of occupations are classified as college occupations. With this definition in hand, we update the empirical moments, re-calibrate the model, and find that unobserved heterogeneity explains 92.96% of the duration dependence observed in the data. This is similar to our baseline result of 95.27%.

Next, we follow Barnichon and Zylberberg (2019) in using the 2012 Occupation Outlook Handbook published by the Bureau of Labor Statistics to measure education requirements. The handbook lists the typical education needed for entry into each occupation. We define a college occupation as one in which a bachelors degree or above is typically required for entry. It is worth noting that 93.6% of occupational classifications (non-college vs. college) are the same as our baseline definition using O*NET. It is not surprising then that we find similar results. Namely, we find that unobserved heterogeneity explains 94.02% of the duration dependence. Appendix B shows the details behind the alternative definitions of non-college and college occupations, the empirical moments, calibrations, and full set of quantitative results under each definition.

The next robustness check uses estimates from Dinerstein et al. (2022) on the rate of skill depreciation and returns to experience instead of relying on the identifying restriction to calibrate the skill accumulation and loss parameters during underemployment. To summarize, Dinerstein et al. (2022) estimate an annual skill depreciation rate of 4.2% and net returns to working on productivity of 2.5%. After setting the skill loss and accumulation parameters, $d_{\chi,\tau}$ for $\chi \in X$, so that the model is consistent with the evidence from Dinerstein et al. (2022) and calibrating the rest of the parameters as in Section 5.1, we find that unobserved heterogeneity explains 94.99% of duration dependence in underemployment. Appendix C.4.1 contains more particulars about this exercise.

Finally, recall that workers compensate firms for their recruiting costs through a one-time hiring fee. While our baseline analysis uses the wages under this contract to discipline the skill loss and accumulation parameters, we can also use the worker's effective wage. That is, the wage workers would earn if they instead paid a per-period fee to the firm. Suppose that the worker pays a fee $\hat{\xi}_\chi(v, \tau)$ to the firm each period, where the present discounted value of the payments is equivalent to a one-time fee paid at the beginning of the match. The effective wage is $\hat{w}_\chi(v, \tau) = y_\chi(v, \tau) - \hat{\xi}_\chi(v, \tau)$. With the effective wages in hand, we re-calibrate the model and find that unobserved heterogeneity explains 93.54% of the duration dependence in underemployment. The specifics of the effective wages, calibration, and decomposition exercise can be found in Appendix C.4.2.

6 Conclusion

This paper has studied underemployment durations among recent college graduates in the US. Using the NLSY97, we have shown that the probability a worker exits underemployment decreases in their underemployment duration and that longer underemployment histories are associated with lower wages in college occupations.

To explain these facts, we developed a directed search model with unobserved heterogeneity, occupation-specific human capital, and on the job search. Workers learn about their job-finding probability in college jobs through search. Underemployment is generated when workers with a low expected suitability self-select into non-college jobs. Underemployed workers face both the accumulation of non-college and decay of college skills, creating structural duration dependence in underemployment. A quantitative analysis shows that unobserved heterogeneity is a large source of both the existence of underemployment and the duration dependence observed in the data.

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Supplementary Materials

A Empirical Appendix

A.1 National Longitudinal Survey of Youth 1997 (NLSY97)

The NLSY97 tracks the lives of 8,984 individuals born between 1980 and 1984. It covers employment activities that can affect the ability to obtain and perform a job (such as education, training, etc.), as well as other sections on marriage, fertility, household composition, and health. The survey was on an annual basis from 1997 through 2011 and biennially thereafter. All respondents were ages 12 to 17 at their first interview. As our analysis requires observing consecutive employment records, we do not use any post-2011 employment records.

Our sample construction begins with selecting individuals who obtained a bachelors degree or above and have at least one year of labor market experience before 2011. As the analysis of duration dependence requires consecutive employment records, 171 out of 1,974 college graduates with at least one year of an employment record are dropped because of missing employment records in some weeks and 651 are dropped because they have a missing occupation code. Next, we drop 6 respondents who are always enrolled in school after obtaining a college degree. Finally, all employment records start from when the worker completely enters the labor market, leaving 996 respondents with at least one year of employment records after their last enrollment in school. Table A1 provides summary statistics.

The complete employment history includes weekly working hours, employment status, 2002 Census Industry and Occupation Codes, and real hourly wage.[23] The individual characteristics we focus on are gender, age, race, Armed Services Vocational Aptitude Battery (ASVAB) percentile, education history (including the highest education and the graduation date), college major, college GPA, student loan debt, and family income per-capita. As for the college major, we focus on primary majors self-reported term by term, and consider the major reported the most times during the undergraduate period as an individual's major. In terms of college GPA, we take the average GPA across all available reported terms. Finally, the financial information has also been extracted to identify the amount of student loan currently owed and the family income per-capita, where the latter is calculated by dividing the total family income by the household size. We also consider

[23]We deflate the hourly rate of pay with the Consumer Price Index. We then code the deflated hourly pay rate to missing if that is less than $1 or more than $1,000. Additionally, we code weekly working hours to missing if less than 10 or more than 98 hours. The average working hours per week among underemployed (properly employed) workers is 42.97 (45.25).

the possibility of dual jobs and find that less than 13% of observations have more than one job. For these observations, we code the main job to be the one with the highest real wage.

Table A1: Descriptive Statistics

	N	Mean	Std. Dev	Min	Max
Gender	996	0.56	0.50	0	1
– Male	439	0	-	-	-
– Female	557	1	-	-	-
Birth year	996	1982.06	1.40	1980	1984
– 1980	176	1980	-	-	-
– 1981	207	1981	-	-	-
– 1982	202	1982	-	-	-
– 1983	204	1983	-	-	-
– 1984	207	1984	-	-	-
Race	996	3.24	1.19	1	4
– Black	157	1	-	-	-
– Hispanic	135	2	-	-	-
– Mixed race (Non-Hispanic)	14	3	-	-	-
– Non-Black / Non-Hispanic	690	4	-	-	-
ASVAB percentile	882	69.25	23.11	3	100
Highest degree	1,112	4.42	0.73	4	7
– BA	740	4	-	-	-
– MA	214	5	-	-	-
– PhD	9	6	-	-	-
– Professional degree	33	7	-	-	-
Major	994	0.34	0.475	0	1
– Arts and Social Sciences	695	0	-	-	-
– STEM	299	1	-	-	-
Weekly hours	206,177	44.28	11.30	10	98
Real hourly wage ($)	206,872	17.61	21.59	1.01	519.65
Potential experience (months)	232,953	42.92	26.39	1	127
Student loan currently owed ($K)	232,953	0.26	2.60	0	120
Family income per-capita ($K)	212,420	37.71	38.62	0	421.37
College GPA	232,661	3.23	0.41	1.88	5
College occupation	214,029	0.569	0.495	0	1

A.2 Occupational Information Network (O*NET)

The O*NET measures occupational requirements and worker attributes. It is composed of four survey questionnaires covering skills, knowledge, generalized work activities, and work context. Respondents include job incumbents and occupational experts at various business work sites. Notably, O*NET reports the required level of education to perform a job under the domain of worker requirements, which enables us to determine whether an occupation is one that typically requires a bachelors degree and above.

A.3 Measurement of Occupation Skill Requirements

To measure the distance in skill requirements between occupations, we start by measuring the occupation's skill requirements along three dimensions. Specifically, each occupation is represented by a three-dimensional vector $(r_{verbal}, r_{math}, r_{social})$ where r_{verbal} measures the occupation's verbal skill requirement, r_{math} measures the math/quantitative skill requirement, and r_{social} captures the social skill requirement.

To measure verbal and mathematical skill requirements, we strictly follow the methodology used by Guvenen et al. (2020). The first step is to construct four scores for each occupation. The scores are: (i) word knowledge, (ii) paragraph comprehension, (iii) arithmetic reasoning, and (iv) mathematics knowledge. To construct these scores, we first select 26 O*NET descriptors that are chosen by the Defense Manpower Data Center (DMDC) and are listed in the top of Table A2. In the raw data, these descriptors range in value from 0 to 5. We re-scale their values in each year to fall between 0 and 1 and then take the average value for each descriptor between 2003 and 2011. Finally, we construct a weighted average in each of the four skill categories using the weights matrix provided by the DMDC. For example, to construct the word knowledge score in occupation o, $S_{o,wk}$, we compute

$$S_{o,wk} = \sum_{i=1}^{26} s_{o,i} * \omega_{wk,i}, \tag{A.13}$$

where $s_{o,i}$ is descriptor i's average value between 2003 and 2011 for occupation o and $\omega_{wk,i}$ is the weight given to descriptor i in the category of word knowledge.

Second, we normalize the standard deviation of each score to one and reduce these four scores into two composite indicators, r_{verbal} and r_{math}, by applying principal component analysis (PCA). The verbal skill is the first principal component of word knowledge and paragraph comprehension, and the math skill is the first principal component of arithmetic reasoning and mathematics knowledge. The verbal and math skills are then converted into percentile ranks among all occupations.

Table A2: List of Descriptors

Panel A: Verbal and Math Skills		
Oral Comprehension	Written Comprehension	Deductive Reasoning
Inductive Reasoning	Information Ordering	Mathematical Reasoning
Number Facility	Reading Comprehension	Mathematics Skill
Science	Technology Design	Equipment Selection
Installation	Operation and Control	Equipment Maintenance
Troubleshooting	Repairing	Computers and Electronics
Engineering and Technology	Building and Construction	Mechanical
Mathematics Knowledge	Physics	Chemistry
Biology	English Language	
Panel B: Social Skills		
Social Perceptiveness	Coordination	Persuasion
Negotiation	Instructing	Service Orientation

The social skill requirement can be identified similarly. By applying PCA to six scaled O*NET descriptors, we construct a single index to reflect the social skill requirement and then apply the percentile transformation as described above. The six descriptors used to construct the social skill requirement are listed at the bottom of Table A2. Based on the skill requirement along each dimension $(r_{verbal}, r_{math}, r_{social})$, we proceed to calculate the average skill requirement for each occupation by taking the unweighted average across the three dimensions.

Next, we examine the relationship between skill and education requirements. Table A3 lists the mean skill and education requirements of the five most common college and non-college jobs in our sample. College jobs are associated with higher skill requirements along each skill dimension, as well as the average skill requirement. Figure A.1 further demonstrates by plotting the average skill requirement among non-college and college jobs for verbal, math, social and average skill requirements. The same pattern emerges. In particular, college jobs have significantly higher skill requirements.

Finally, Figure A.2 presents a heat map demonstrating the correlation between skill and education requirements. Darker shades of red indicate a stronger positive correlation. The first column represents the percentage of respondents in the O*NET surveys who state that a bachelors degree or higher is needed to perform a certain occupation. The second column is a binary variable that indicates whether more than 50% of respondents

Table A3: Skill Requirements of Five Most Common College/Non-college Jobs

Occupation	Verbal	Math	Social	Avg.
Panel A: College jobs				
Elementary and middle school teachers	0.82	0.77	0.85	0.81
Registered nurses	0.76	0.67	0.80	0.74
Accountants and auditors	0.64	0.86	0.33	0.61
Secondary school teachers	0.84	0.81	0.92	0.85
Social workers	0.24	0.14	0.96	0.44
Panel B: Non-college jobs				
First-line supervisors/Managers of retail sales workers	0.33	0.44	0.56	0.44
Retail salespersons	0.10	0.21	0.20	0.17
Sales representatives, wholesale and manufacturing	0.28	0.37	0.67	0.44
Secretaries and administrative assistants	0.40	0.23	0.18	0.27
Customer service representatives	0.34	0.31	0.30	0.32

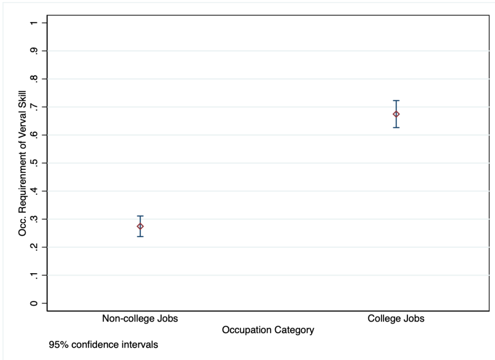
indicate that a bachelors degree or higher is necessary to perform the occupation. Notably, it shows a positive correlation between education and skill requirements.

A.4 Within-firm Transitions

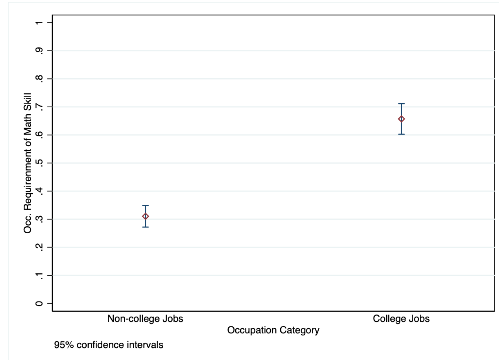
As discussed in the main text, there may be measurement error in within-firm occupational transitions. We attempt to correct this error by identifying "genuine" within-firm occupation switches from non-college to college occupations. To do so, we first measure the angular distance between the skill requirement of the new college occupation and the previous non-college occupation. Specifically, let $\phi\colon \mathbb{R}^3 \times \mathbb{R}^3 \to [0, \pi/2]$, and define the angular distance between two skill vectors $\mathbf{r}_i$ and $\mathbf{r}_j$ as

$$\phi(\mathbf{r}_i, \mathbf{r}_j) = \cos^{-1}\left(\frac{\mathbf{r}_i \cdot \mathbf{r}_j'}{\|\mathbf{r}_i\|\|\mathbf{r}_j\|}\right). \tag{A.14}$$

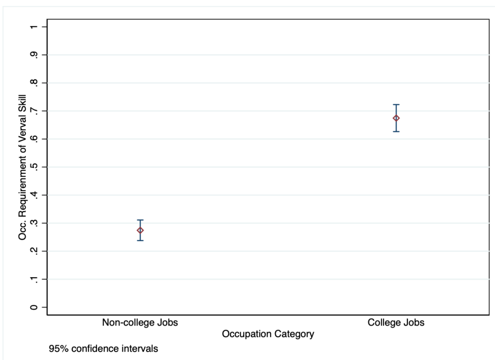
A within-firm transition from an occupation i to another different occupation j is treated as a "genuine" transition if and only if $\phi(\mathbf{r}_i, \mathbf{r}_j) \geqslant \bar{\phi}$ where $\bar{\phi}$ is chosen so that the average correlation in skill requirements in "genuine" switches is close to zero. We set $\bar{\phi} = 18.094$, which results in a correlation in skill requirements among within-firm occupation switches of 0.0048. In our sample, 46/96, or 48% of within-firm transitions from non-college to college jobs are identified as genuine switches, which is close to the



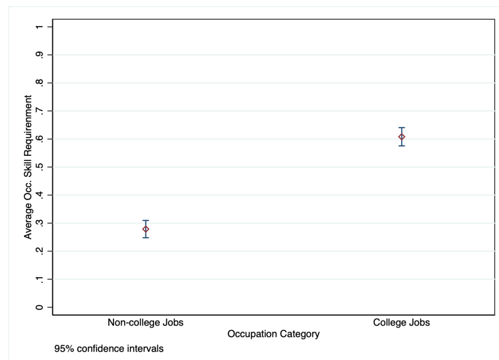
(a) Verbal



(b) Math



(c) Social



(d) Average

Figure A.1: Comparison of Skill Requirements

Notes: Graph shows 95% confidence intervals. We test the null hypothesis that the verbal/math/social/average skill requirement of non-college jobs is the same as that of college jobs against the alternative that the skill requirement of non-college jobs is below that of college jobs, and the test yields a p-value less than 0.01.

propensity of switching careers, 42.1%, obtained by Baley et al. (2022).

An alternative measure of distance between skill requirements is the Euclidean distance, which not only captures the composition of skill requirements, but also the magnitude of each skill requirement. The Euclidean distance between occupation i and j is given by

$$\psi(\mathbf{r}_i, \mathbf{r}_j) = \sqrt{\sum_{k=1}^{3} (r_{i,k} - r_{j,k})^2}, \tag{A.15}$$

where $r_{i,k}$ is occupation i's requirement in aptitude $k \in \{verbal, math, social\}$.

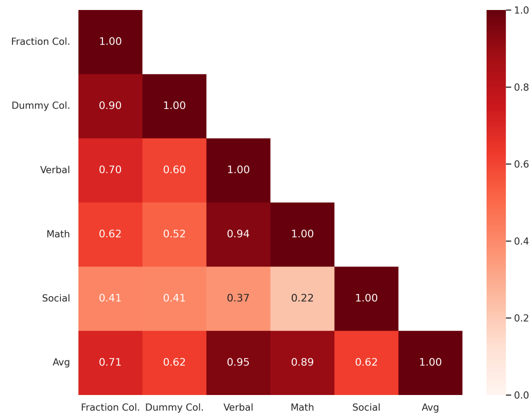


Figure A.2: Correlation Between Education and Skill Requirements

A.5 Skill Distance, College Wages, and Underemployment History

To further support the notion of the accumulation and decay of occupation-specific human capital, we study how the association between college wages and underemployment history varies with the distance in skill requirements between a worker's current college occupation and previous non-college occupation. The idea here is that if the distance in required skills between the two occupations is larger, then the skills required by the new college occupation would have been used less intensively in the previous non-college occupation and thus experienced a greater rate of decay, ultimately leading to a stronger association between underemployment history and wages in college occupations. To assess this hypothesis, we estimate the following regression:

$$w_{it} = \alpha \text{Underhis}_{it} + \gamma\phi_{it} + \zeta \text{Underhis}_{it} \times \phi_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \tag{A.16}$$

where ϕ_{it} is the distance in skill requirements between individual i's current college occupation and their most recent non-college occupation and X contains the same controls as in equation (2) in the main text. We use two measures of distance. The first is the Euclidean distance while the second is the angular distance as in Baley et al. (2022). The estimation of (A.16) only includes observations among individuals currently employed in a college occupation and who have been previously underemployed. Moreover, we restrict to those individuals where the average skill level in their current college occupation is higher than their previous non-college occupation.

Table A4 contains the results. Column (1) reveals that a larger Euclidean distance is associated with a significantly stronger relationship between the worker's underemploy-

Table A4: Skill Distance, College Wages, and Underemployment History

	(1)	(2)
Underhis	0.0173	0.0374***
	(0.0106)	(0.0098)
Underhis×Euclidean distance	-0.0454***	
	(0.0103)	
Underhis×Angular distance		-0.0017***
		(0.0004)
N	16,594	16,594
R^2	0.924	0.923

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

ment history and wages in college occupations. Column (2) echoes this result when we use the angular distance.

A.6 Duration Dependence with Different Sets of Control Variables

To elucidate the factors that attenuate the duration dependence profile by controlling for observables, we take a closer look at the estimation in (2) by progressively incorporating control variables as follows:

$$
\begin{aligned}
y_{it} = {} & \beta\tau_{it} + \delta_t + \epsilon_{it} && \text{(Control 1)} \\
& + \text{Gender}_i + \text{Race}_i + \text{Edu}_i + \text{Gender}_i \times \text{Race}_i + \text{Gender}_i \times \text{Edu}_i + \text{Age}_{it} + \text{ASVAB}_i && \text{(Control 2)} \\
& + \text{Major}_i + \text{GPA}_i && \text{(Control 3)} \\
& + \text{FamInc}_{it} + \text{Loan}_{it} && \text{(Control 4)} \\
& + \text{JobSat}_{it}. && \text{(Control 5)}
\end{aligned}
$$

Figure A.3 presents the results. The red curve (Control Set 1) illustrates the transition path when controlling for year and month fixed effects. Control Set 2 additionally controls for gender, race, highest education, gender interacted with race, gender interacted with highest education, age bins and ASVAB scores, that produces the orange curve. We then add college major (Arts and Social Science versus STEM) and GPA bins, producing the yellow line under Control Set 3. By further controlling for family income per-capita and student loan owed, the green curve represents the duration dependence under Control

Set 4. Finally, the blue curve (Control Set 5) reveals a notable attenuation in duration dependence profile when we control for the current job satisfaction.

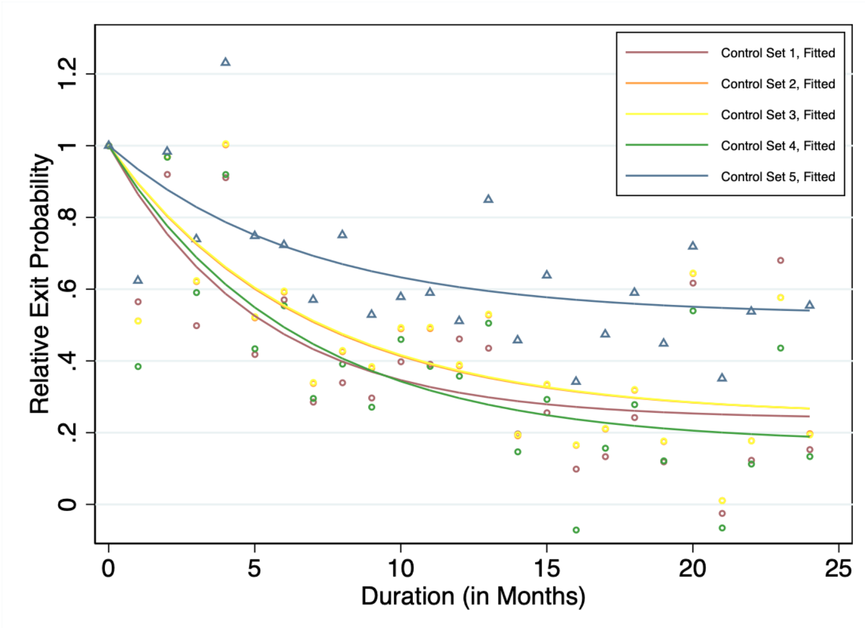


Figure A.3: Attenuation of the Duration Dependence Profile

A.7 Supplementary Tables and Figures

Table A5: Occupations within 5 Percentage Points of the 50% Threshold

Occupation Title	College Fraction
Geological and petroleum technicians	46.19
Other life, physical, and social science technicians	48.72
Sales representatives, wholesale and manufacturing	49.76
Designers	50.13
Directors, religious activities and education	50.97
Religious workers, all other	50.97
Cost estimators	51.03
Producers and directors	51.57
Construction managers	52.10
Judges, magistrates, and other judicial workers	53.40
Writers and authors	53.82
Other business operations specialists	54.15
Network systems and data communications analysts	54.49

Table A6: Top 10 College and Non-college Occupations

College Occupations	N	Non-college Occupations	N
Elementary/Middle school teachers	11,771	First-Line supervisors/Managers of retail sales workers	6,788
Registered nurses	5,990	Retail salespersons	3,806
Accountants and auditors	5,762	Sales representatives, wholesale and manufacturing	3,297
Secondary school teachers	5,761	Secretaries and administrative assistants	3,136
Social workers	4,703	Customer service representatives	2,978
Managers, all other	3,989	Police and sheriff's patrol officers	2,973
Financial managers	3,559	First-Line supervisors/Managers of office and admin. support workers	2,692
Other teachers and instructors	3,517	Waiters and waitresses	2,481
Computer software engineers	3,396	Cashiers	1,879
Marketing and sales managers	3,225	Loan counselors and officers	1,866

Table A7: Underemployment and College Major

Major	*N*	Respondents	Underemp. ratio
A: Arts and Social Sciences			
Liberal arts and science	104	2	0.337
International relations and affairs	156	1	0.122
Social work	187	1	0.989
Archaeology	291	1	0.808
Hotel/Hospitality management	500	3	0.790
Pre-law	531	2	0.452
Human services, general	578	3	0.351
Home economics	595	4	0.606
Area studies	709	2	0.234
Anthropology	709	6	0.068
Theology/Religious studies	1,148	5	0.462
Philosophy	1,361	5	0.505
Foreign languages	2,244	8	0.311
English	4,786	24	0.335
Political science and government	5,422	26	0.375
Economics	5,636	16	0.265
History	5,832	32	0.482
Sociology	6,905	31	0.283
Criminology	7,022	31	0.573
Fine and applied arts	11,928	45	0.635
Psychology	16,015	69	0.398
Communications	17,910	68	0.442
Education	20,574	99	0.242
Business management	56,538	211	0.484
All Arts and Social Sciences	167,681	695	**0.429**
B: STEM			
Pre-vet	156	1	0.865
Nutrition/Dietetics	365	2	0.399
Pre-med	448	4	0.213
Agriculture/Natural resources	2,163	8	0.626
Mathematics	2,621	13	0.491
Interdisciplinary studies	2,622	12	0.387
Physical sciences	3,131	16	0.262
Architecture/Environmental design	3,132	15	0.213
Nursing	6,378	28	0.035
Other health professions	7,982	38	0.393
Biological sciences	10,430	49	0.251
Computer/Information science	12,634	52	0.446
Engineering	13,096	61	0.312
All STEM	65,158	299	**0.325**

Notes: To compute the average underemployment ratio for Arts and Social Sciences and STEM, each major's ratio is weighted by its respective number of observations.

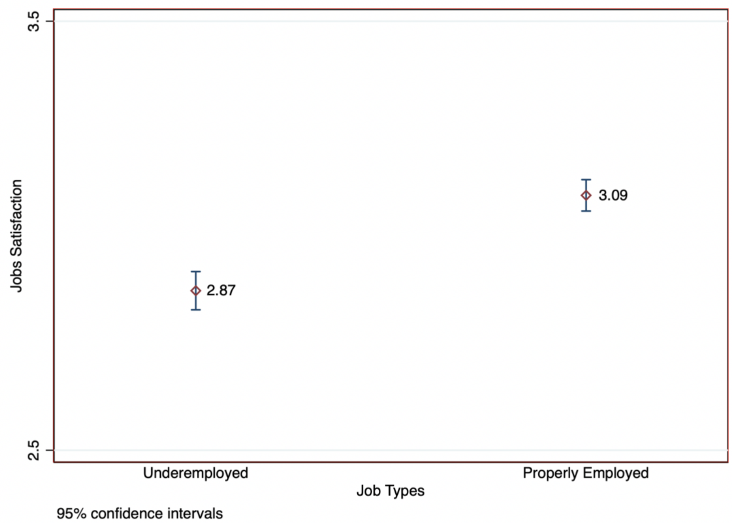
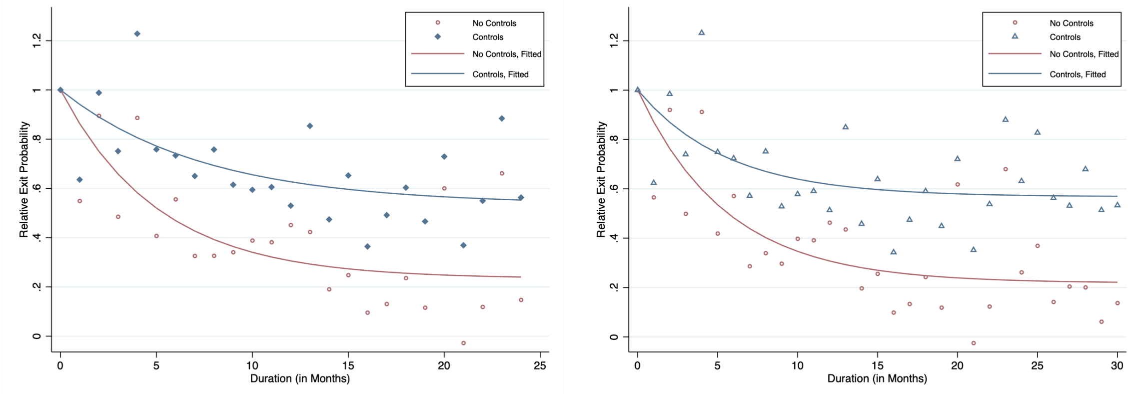


Figure A.4: Job Satisfaction among Underemployed and Properly Employed

Notes: We test the null hypothesis that the job satisfaction is equal among the underemployed versus the properly employed against the alternative that the job satisfaction among the properly employed is different from the job satisfaction among the underemployed. The p-value is less than 0.01, indicating significant differences in job satisfaction between the two groups.



(a) Extra Transition Time

(b) Capped at 30 Months

Figure A.5: Additional Duration Dependence Profiles

Table A8: College Wages and Underemployment History (1-digit Industry and Occupation Codes)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0166***		-0.0164***	-0.0148***		-0.0145***
	(0.0010)		(0.0010)	(0.0011)		(0.0010)
Underhis		0.0006***	0.0006***		0.0010***	0.0009***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College $\times$ Unhis				-0.0011		-0.0002
				(0.0013)		(0.0013)
College $\times$ Underhis					-0.0023***	-0.0022***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.778	0.777	0.778	0.774	0.774	0.774

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), regional and national annual unemployment rates, age, age squared, per-capita family income ($\$K$), student loan debt ($\$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 1-digit level), and region.

Table A9: College Wages and Underemployment History (1-digit Occupation FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0154***		-0.0153***	-0.0153***		-0.0150***
	(0.0010)		(0.0010)	(0.0011)		(0.0011)
Underhis		0.0004***	0.0004***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College $\times$ Unhis				0.0002		0.0008
				(0.0014)		(0.0014)
College $\times$ Underhis					-0.0022***	-0.0021***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172,149	172,149	172,149	172,149	172149	172149
R^2	0.784	0.784	0.784	0.784	0.784	0.785

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), regional and national annual unemployment rates, age, age squared, per-capita family income ($\$K$), student loan debt ($\$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 1-digit level), and region.

Table A10: College Wages and Underemployment History (2-digit Occupation FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0145***		-0.0145***	-0.0144***		-0.0141***
	(0.0009)		(0.0009)	(0.0010)		(0.0010)
Underhis		0.0003***	0.0003***		0.0005***	0.0005***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × Unhis				-0.0004		-0.0000
				(0.0013)		(0.0013)
College × Underhis					-0.0017***	-0.0016***
					(0.0001)	(0.0001)
2-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.791	0.790	0.791	0.791	0.791	0.791

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), regional and national annual unemployment rates, age, age squared, per-capita family income ($\$K$), student loan debt ($\$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and region.

Table A11: College Wages and Underemployment History (Year and Month FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0144***		-0.0143***	-0.0137***		-0.0134***
	(0.0009)		(0.0009)	(0.0011)		(0.0011)
Underhis		0.0003***	0.0003***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × Unhis				-0.0012		-0.0007
				(0.0013)		(0.0013)
College × Underhis					-0.0020***	-0.0019***
					(0.0002)	(0.0002)
2-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.792	0.791	0.792	0.783	0.783	0.784

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables including the original, squared, and cubic potential experience (in months), regional and national annual unemployment rates, age, age squared, per-capita family income ($\$K$), student loan debt ($\$K$), and job satisfaction. Furthermore, the analysis includes fixed effects for calendar year, calendar month, individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and region.

A.8 Information Frictions and Underemployment

This appendix presents suggestive evidence that college graduates have uncertainty regarding their abilities, learn about their type over time, and that information frictions are a source of underemployment. To begin, we first construct a proxy measure of information frictions by leveraging a set of questions in the National Longitudinal Survey of Youth 1979 (NLSY79). The NLSY79 asks the following questions to all respondents, one time, in their initial interview:

(i) What kind of work do you think you would be doing 5 years from now? If more than one occupation, what one kind of work do you think you would prefer?

(ii) What kind of work would you like to be doing when you are 35 years old?

Unfortunately, these questions were not asked in the NLSY97, our main dataset. To conduct this analysis on the NLSY79 cohort, we apply the same sample selection criteria that is applied to the NLSY97 sample, which leaves 444 college graduates with both a valid response to the 5 year horizon question and who were employed five years later, and 1,006 with a response to the occupational expectation at age 35 and identifiable realized occupation at age 35.

To construct a proxy measure of information frictions, we compare the skill requirements in a worker's realized occupation and their anticipated occupation, both at 5 years after the initial survey (short-term forecast) and when the respondent is 35 years old (long-term forecast). The occupational forecast error is given by the difference in required skills in aptitude $j \in \{v, m, s\}$ (verbal, math, and social) between one's anticipated occupation ($\hat{s}_j$) and the realized occupation (s_j):

$$\overline{FCE}_i = \frac{\sum_{j \in \{v,m,s\}} |s_j - \hat{s}_j|}{3}. \tag{A.17}$$

Figure A.6 displays the distribution of short-term and long-term forecast errors.[24] Of the 444 college graduates, only 74 (or 17%) end up matched with their expected occupation in 5 years. Further, the average short-term forecast error is 0.22. Among the 1,006 graduates with a valid long-term forecast error, 146 (or 15%) are exactly matched to their expected occupation by age 35. Given the longer prediction horizon, the average long-term forecast error is 0.27.

[24]The violin plot illustrates the distribution characteristics of FCE, where the thick bar in the middle represents the interquartile range of FCE. The thin lines extending from it signify the 95% confidence interval, and the white dot denotes the median of FCE.

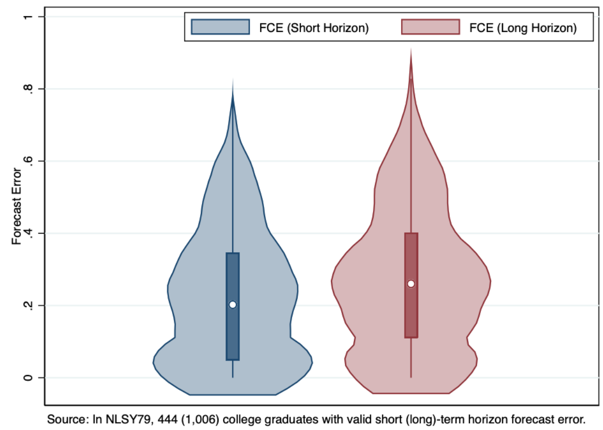


Figure A.6: Distributions of Forecast Errors

In addition to these descriptive findings, we conduct a more formal test for the presence of information frictions, which follows Baley et al. (2022). Using the NLSY79, we define an individual i's forecast error in skill j between the realized occupation and predicted occupation at time $t+\Delta$ as

$$FCE_{i,j,t} \equiv q_{i,j,t+\Delta} - \hat{q}_{i,j,t+\Delta}, \tag{A.18}$$

where t is the date of the initial interview, and Δ could be either 5 years or the duration from the time of the initial interview until the individual reaches the age of 35. Suppose that worker i knows their vector of skills across the j aptitudes, $\mathbf{a_i}$, and that skills are predictive of future occupations. With this in mind, one can predict the forecast error regarding the utilization of skill j by computing

$$\begin{aligned} PE_{i,j,t} &\equiv \mathbb{E}[FCE_{i,j,t}|\mathbf{a_i}], && \text{(A.19)}\\ &= \mathbb{E}[q_{i,j,t+\Delta}|\mathbf{a_i}] - \hat{q}_{i,j,t+\Delta}, && \text{(A.20)}\\ &= a_{i,j} - \hat{q}_{i,j,t+\Delta}. && \text{(A.21)} \end{aligned}$$

As $a_{i,j}$ and $\hat{q}_{i,j,t+\Delta}$ are both realized at the survey time t, the predicted error is realized at time t. Following Chahrour and Ulbricht (2023), the predicted and realized forecast errors are orthogonal to each other, $\text{Corr}(FCE_{i,j,t}, PE_{i,j,t}) = 0$, under the null hypothesis of full information. To examine whether the hypothesis of full information regarding workers' ability is supported by the data, we estimate the following regression:

$$\sum_{j\in v,m,s} FCE_{i,j,t} = \beta_0 + \beta_1 \sum_{j\in v,m,s} PE_{i,j,t} + \epsilon_{i,t}. \tag{A.22}$$

Additionally, we test the hypothesis along each skill aptitude j by estimating

$$FCE_{i,j,t} = \beta_0 + \beta_1 PE_{i,j,t} + \epsilon_{i,j,t}. \tag{A.23}$$

Given that we have data on both short-term and long-term occupational expectations, we can examine the full information hypothesis using different horizons of forecast error. Tables A12 and A13 present the results over a 5-year horizon or at age 35, respectively. In all cases, the coefficients β_1 are statistically significant at the 1% level, which leads us to reject the null hypothesis that workers have full information about their abilities. These findings, which support the presence of incomplete information regarding one's ability, are also well documented in the literature. See, for example, Baley et al. (2022), Guvenen et al. (2020) and Conlon et al. (2018).

Beyond the presence of information frictions, the statistically significant positive β_1 in Tables A12-A13 implies that workers learn their type over time. For example, if a worker underestimates her usage of verbal skill in the future, captured by a positive $PE_{i,t,v}$, our finding suggests that workers gain more certainty about their type, and tend to move towards a more verbal-intensive job than initially anticipated.

Table A12: Testing for Information Frictions (Expected Occupation in Five Years)

		Dependent Variable: $FCE_{i,j,t}$		
	(1)	(2)	(3)	(4)
	$\sum_j FCE_{i,j,t}$	Verbal	Math	Social
$\sum_j PE_{i,j,t}$	0.422***			
	(0.040)			
$PE_{i,j,t}$		0.326***	0.399***	0.387***
		(0.035)	(0.041)	(0.037)
N	444	444	444	444
R^2	0.219	0.168	0.201	0.199

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

To further support information frictions as a source of underemployment, and given that the forecast error is a proxy for the workers' uncertainty regarding their type, we examine whether college graduates who made larger errors in forecasting their future occupation are more likely to end up underemployed upon their entry into the labor market. We estimate the correlation between the magnitude of forecast error and their

Table A13: Testing for Information Frictions (Expected Occupation at Age 35)

		Dependent Variable: $FCE_{i,j,t}$		
	(1)	(2)	(3)	(4)
	$\sum_j FCE_{i,j,t}$	Verbal	Math	Social
$\sum_j PE_{i,j,t}$	0.540***			
	(0.027)			
$PE_{i,j,t}$		0.447***	0.530***	0.401***
		(0.024)	(0.027)	(0.024)
N	1,006	1,006	1,006	1,006
R^2	0.288	0.245	0.269	0.194

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

probability of being underemployed in their first job by estimating

$$Y_i^n = \beta_1 \overline{FCE}_i + \Gamma \cdot X_i + \epsilon_i. \tag{A.24}$$

The dependent variable, Y_i^n, is a dummy indicating if respondent i's first job is a non-college job or not. Alternatively, we use a dummy, Y_i^c, which indicates whether the first job is a college job as the dependent variable. The vector, X, contains gender, race, highest education, the interaction between gender and race, the interaction between gender and highest education, and the average skill level.

Table A14: Forecast Error and Underemployment

	Expectation in 5 Years		Expectation at Age 35	
	Y_i^n	Y_i^c	Y_i^n	Y_i^c
β_1	0.251**	-0.300**	0.148*	-0.184**
	(0.127)	(0.125)	(0.076)	(0.075)
N	444	444	1,006	1,006
R^2	0.044	0.063	0.087	0.097

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

Table A14 presents the results, from which two primary takeaways emerge. First, workers with a higher forecast error are more likely to become underemployed upon initially entering the labor market, supporting the argument that information frictions serve as a source of underemployment. Second, workers with more certainty typically transition directly to employment in college jobs. This aligns with our quantitative result that removing information frictions causes broad-suitability workers to continually search for college jobs, resulting in lower underemployment.

Alternatively, we can explore the correlation between one's forecast error and the incidence of underemployment over the career by estimating

$$Y^n_{i,t}(Y^c_{i,t}) = \beta_1 \overline{FCE}_i + \beta_2 \text{Potexp}_{i,t} + \beta_3 \overline{FCE}_i \times \text{Potexp}_{i,t} + \Gamma \cdot X_i + \text{Month}_t + \text{Year}_t + \epsilon_{i,t}, \tag{A.25}$$

where $Y^n_{i,t}$ ($Y^c_{i,t}$) is a dummy indicating whether worker i is underemployed (properly employed) or not at time t and $\text{Potexp}_{i,t}$ is individual i's potential experience at time t. Equation (A.25) contains month and year fixed effects, in addition to the same individual level controls as in (A.24). In particular, β_1 captures the correlation between the forecast error and the probability of being underemployed, while β_3 reflects how this correlation evolves over one's career.

Table A15: Forecast Error and Underemployment over the Career

	Expectation in 5 Years		Expectation at Age 35	
	$Y^n_{i,t}$	$Y^c_{i,t}$	$Y^n_{i,t}$	$Y^c_{i,t}$
β_1	0.4757***	-0.5525**	0.2769***	-0.3526***
	(0.0127)	(0.0127)	(0.0076)	(0.0078)
β_3	-0.0005***	0.0006***	-0.0002***	0.0003***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
N	130,152	130,152	347,099	347,099
R^2	0.0655	0.0742	0.0723	0.0812

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

Table A15 presents the results using forecast error measured by occupation expectation in 5 years or at age 35. The primary finding is that workers who make a larger forecast error are more (less) likely to be underemployed (properly employed). Moreover, the impact of the forecast error diminishes over the course of one's career, as indicated by the opposite sign of β_3. This supports the notion of learning over one's career. As workers learn about their type over time, the initial uncertainty regarding their suitability becomes less influential.

A.9 Correlation between Unemployment and Underemployment Durations

This section details the estimation of the correlation between the duration of underemployment and its prior unemployment duration. Given that workers can transition from non-college to college jobs multiple times, we calculate the correlation both for all transitions and first transitions. To maintain consistency with the model, we have assumed that (i) unemployment and non-employment are treated as equivalent, (ii) each college graduate enters the labor market with a one-month (or 4 weeks) period of unemployment as workers in the model are unemployed for one period before they can begin searching for a job, and (iii) the unemployment (underemployment) duration is capped at 12 (24) months.

To compute the unconditional correlation, we first identify the unemployment duration before the current underemployment spell. Then, we convert the duration (both prior unemployment and current underemployment duration) in weeks to months by taking each four weeks as one month, and cap the unemployment duration to 12 months and underemployment duration to 24 months. Subsequently, we compute the correlation between the underemployment duration and the previous unemployment duration.

For the conditional correlation, we start by converting weekly employment histories into a monthly basis by determining the primary labor force status for each month. Our criteria for this transformation are as follows: First, the labor force status most frequently reported within a month is regarded as the primary status for that period. Second, if the number of weeks in underemployment is identical to that in any other status (such as unemployment or proper employment), the month is classified as underemployed. Third, if the duration in weeks of unemployment equals that of proper employment, the month is categorized as unemployed. Based on the monthly employment data, we compute the correlation by estimating

$$\begin{aligned}\tau_{ik} = \beta_0 + \beta_1 v_{ik} &+ \text{Gender}_i + \text{Race}_i + \text{Edu}_i + \text{Gender}_i \times \text{Race}_i + \text{Gender}_i \times \text{Edu}_i \\ &+ \text{ASVAB}_i + \text{Major}_i + \text{GPA}_i + \text{AvgAge}_{ik} + \text{AvgFamInc}_{ik} + \text{AvgLoan}_{ik} \\ &+ \text{AvgSat}_{ik} + \text{StartYear}_{ik} + \text{StartMonth}_{ik} + \epsilon_{ik}.\end{aligned} \tag{A.26}$$

The dependent variable is the duration in months of the k^{th} underemployment spell for college graduate i. For the control variables, we consider gender, race, education, interactions of gender with race and education, ASVAB score in bins, college major, GPA in bins, average age, average family income, average outstanding student loan debt, average job satisfaction, the start year and month of the k^{th} underemployment spell.

A.10 More on Wages and Underemployment

This section takes a closer look into the relationship between underemployment and wages in college jobs. We estimate the wage path of college jobs as a function of underemployment histories by

$$w_{it} = \alpha + \sum_{\tau=1}^{24} \beta_\tau \times \mathbb{1}(\text{Underhis}_{it} = \tau) + \Gamma \cdot X_{it} + \delta_i + \epsilon_{it}, \tag{A.27}$$

where w_{it} is log wage in college jobs in time t. The right-hand side of (A.27) contains a series of dummy variables equal to 1 if the worker's underemployment history is equal to τ months for $\tau \in \{1, 2, \ldots, 24\}$, an individual fixed effect, and a vector of controls, X_{it}, that contains the same controls as in equation (3). Note that the college observations with $\tau = 0$ serves as the baseline, and thus β_τ measures the effect of month τ^{th} of underemployment on wages in college jobs, relative to the effect of having zero months of underemployment history.

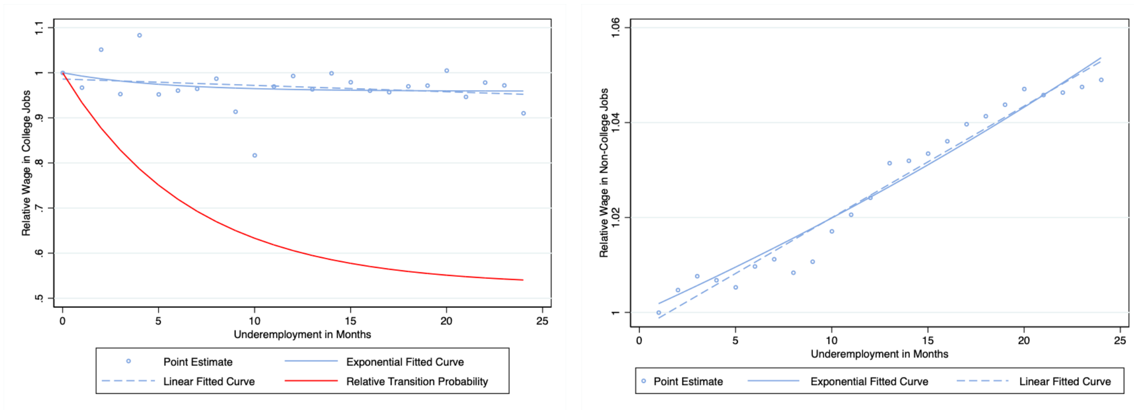
Figure 7(a) plots the estimated coefficients for β_τ (the blue circles). The dashed line shows the linear fit through the β_τ coefficients whereas the solid blue line represents the curve which is generated by estimating the following negative exponential model via weighted nonlinear least squares:

$$f(\tau) = a_1 + (1 - a_1)\exp(-b_1 \tau). \tag{A.28}$$

In (A.28), $f(\tau)$ is the relative wage at underemployment history τ to a worker with underemployment history $\tau = 0$, i.e. the coefficient of β_τ. Note that this model is the same as (1), and the solid red line in Figure A.7(a) reproduces the relative exist probabilities from underemployment to proper employment originally shown in Figure 1.

Figure A.7(b) presents the results from estimating the same regression specified by (A.27), except with wages in non-college jobs as the dependent variable. We also present the linear and exponential fitted patterns through the β_τ coefficients.

As a second exercise, we evaluate the effect of each month of underemployment his-



(a) College Jobs (b) Non-College Jobs

Figure A.7: Wages and Underemployment History

tory on wages in college jobs relative to non-college jobs by estimating:

$$
\begin{aligned}
w_{it} = & \sum_j \beta_j^n \times \mathbb{1}(\text{Unhis}_{it} = j) + \sum_k \beta_k^n \times \mathbb{1}(\text{Underhis}_{it} = k) + \sum_j \beta_j^n \times \mathbb{1}(\text{Unhis}_{it} = j) \times \text{College}_{it} \\
& + \sum_k \beta_k^c \times \mathbb{1}(\text{Underhis}_{it} = k) \times \text{College}_{it} + \text{College}_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}.
\end{aligned}
\tag{A.29}
$$

The dependent variable in (A.29) is individual i's log wage in time t, College is a dummy for whether individual i is employed in a college occupation, δ_i is an individual fixed effect, and the vector X contains the same controls as in (3) and (A.27). Equation (A.29) also includes dummies for each unemployment and underemployment history.

We are particularly interested in the coefficients β_k^c, as these capture the effect of the k^{th} month of underemployment on wages in college jobs relative to non-college jobs. Figure A.8 presents the results from estimating (A.29) with $j \in \{0, 1, 2, \ldots 9\}$ and $k \in \{1, 2, \ldots, 60\}$. The scatter points represent the estimated coefficients, β_k^c, while the lines represent several fits through the scatter points. The blue line is the linear fit, the light-brown line is a quadratic fit, and the green line is a cubic fit. Finally, the red line is the result of estimating a locally weighted regression of β_k^c on the underemployment history, k.[25]

[25]The locally weighted fitted curve is obtained using the default code *lowess*, which creates the fitted curve by using the data points (x_i, y_i) and its nearby data. Lowess is desirable for its locality, enabling it to closely track the data.

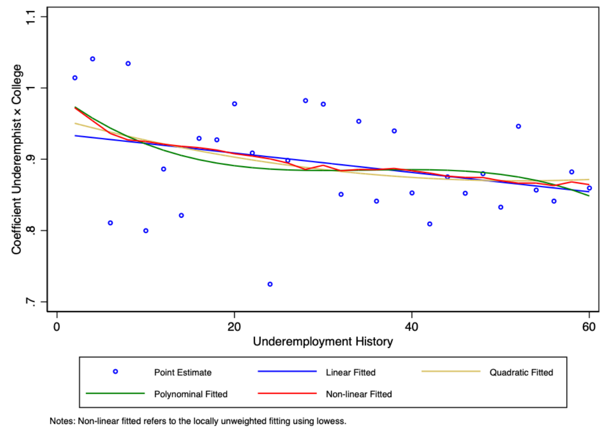


Figure A.8: Linear and Non-Linear Fits through Estimates of β_k^c Coefficients

A.11 Suggestive Evidence on Unobserved Heterogeneity

The results in Section 5 suggest that unobserved heterogeneity in workers' suitability for college jobs plays a large role in generating duration dependence in underemployment. In this section, we provide two sources of suggestive evidence from hourly wage data in the NLSY to support this finding. While wages are not a function of a suitability in the baseline theory, our model has several natural implications for comparing wages in college occupations between workers who experience short- and long-underemployment spells.

First, suppose that wages were a function of a worker's suitability type. This would occur, for example, if limited suitability workers produced less output than broad-suitable workers. If this were the case, then the amount of residual wage dispersion among workers who transition from non-college to college jobs at relatively short underemployment durations would be larger than the amount of dispersion among the group who take longer to transition out of underemployment. In other words, there should be more wage inequality among observationally equivalent workers in the former group than the latter because the latter is primarily comprised of limited suitability workers. To investigate this, we estimate residual wage inequality in jobs held after a worker exits underemployment. The approach, which follows Acemoglu (2002), starts by estimating

$$w_{it} = \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \tag{A.30}$$

where w_{it} is individual i's log hourly real wage at time t and X is a set of controls that includes years of potential experience (original, quadratic and cubic), the annual national unemployment rate, the annual regional unemployment rate, age (original and

Table A16: Residual Wage Dispersion

	90/50 ratio	50/10 ratio
Full sample	1.525	1.568
A: After the first underemployment spell		
$<$ 1 year to exit underemployment	1.324	1.306
$\geqslant$ 1 year to exit underemployment	1.201	1.196
B: Proper employment spell following underemployment		
$<$ 1 year to exit underemployment	1.319	1.315
$\geqslant$ 1 year to exit underemployment	1.237	1.238

quadratic), family income per-capita, outstanding student loan debt, current level of job satisfaction, region, and 2-digit occupation and industry fixed effects. Finally, δ_i is an individual fixed effect. After estimating (A.30), we compute the ratio of the 90th to 50th and 50th to 10th percentile of the residuals.

The first row of Table A16 shows the 90/50 and 50/10 ratios for our entire sample. We see that the 90/50 ratio is 1.53 and the 50/10 ratio is 1.57, which is slightly below the typical range of 1.7-1.9 (Hornstein et al., 2011). Panel A restricts the sample to wages earned in college jobs following a worker's first underemployment spell and shows that the 90/50 (50/10) ratio is 12% (11%) larger in the group that exits underemployment in less than one year. Panel B shows that this pattern also emerges when we focus on the proper employment spell which immediately follows a spell of underemployment.

Our second form of evidence compares wage growth in college jobs between workers who experience short and long underemployment spells. The intuition is the following. If broad-suitable workers have a higher ability to learn new skills, then wage growth in college jobs should be higher among the group of workers who experience short underemployment durations, as this group contains most of the broad-suitable workers who experience underemployment. To investigate this, we estimate the following regression:

$$\Delta w_{it} = \beta \text{Long}_{it} + \Gamma \cdot X_{it} + \varepsilon_{it}, \tag{A.31}$$

where Δw_{it} is the difference in the log of average hourly real wage between quarter $t-1$ and quarter t in the worker's first proper employment spell and Long is equal to 1 if the worker's first underemployment spell lasted a year or longer and 0 otherwise. The vector X includes a cubic in potential experience, highest level of education, race, gender,

Table A17: Wage Growth After Exiting Underemployment

	(1)	(2)	(3)	(4)
Long	-0.0233**	-0.0230**	-0.0224**	-0.0233**
	(0.0091)	(0.0091)	(0.0096)	(0.0099)
N	1,815	1,815	1,779	1,779
R^2	0.025	0.029	0.029	0.030

Notes: Standard errors are clustered at the individual level. The first specification consider a cubic in potential experience, highest level of education, race, gender, age, age square, as well as yearly, 2-digit occupation and industry fixed effects. The second specification additionally controls for the one-year lagged unemployment rate. The third specification additionally controls for the marital status. The last specification additionally consider the interaction between the marital status and age, and its interaction with gender. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

age, age square, and 2-digit occupation and industry fixed effects. Table A17 contains the results and shows, across all specifications, workers who experience longer underemployment spells exhibit lower wage growth in college jobs than their observationally equivalent peers.

B Sensitivity Tests: Alternative Definitions of College Jobs

In the baseline analysis, college jobs are defined as those occupations where at least 50% of respondents indicate that a bachelors degree or above is required to perform the job. For brevity, we label this as the "ONET50.00" definition. To ensure that our primary findings are not sensitive to a particular threshold, we explore alternative criteria for identifying college jobs.

B.1 Alternative Definitions

Our first alternative uses the O*NET descriptors but adopts a lower threshold. Specifically, we redefine college jobs as those occupations in which at least 42.27% of respondents indicate a bachelors degree or above is necessary. This definition, henceforth referred to as the "ONET42.27", corresponds to the 60^{th} percentile in the empirical cumulative density function of college fraction across 298 distinct occupations, as depicted in Figure B.1. Under the ONET42.27 definition, more occupations are classified as college jobs. Such an expansion could lead to more transitions from underemployment to proper employment, potentially attenuating the observed magnitude of duration dependence. Under the ONET42.27 definition, 120 occupations are identified as college jobs, compared to 108 college occupations under the ONET50.00 criteria.

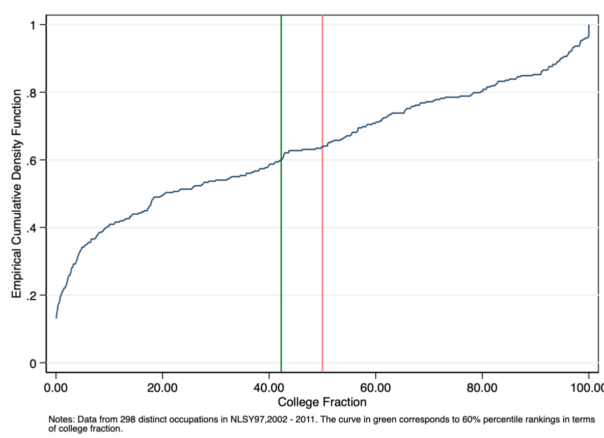


Figure B.1: Empirical CDF of College Fraction

The second approach, which follows Barnichon and Zylberberg (2019), employs the variable *typical education needed for entry* reported by the 2012 Occupation Outlook Handbook, issued by the U.S. Bureau of Labor Statistics, henceforth labelled as the "OOH2012" definition. In particular, the 2012 Occupation Outlook Handbook details the typical entry education for 820 distinct SOC2010 occupations from federal and state regulations and from the typical path of entry into a job.[26] Under this definition, an occupation is considered to be a college job if its typical education needed for entry is a bachelors degree or above.

To identify the typical entry education for each SOC2002 occupation in our sample, we convert the occupation codes from the SOC2010 (used in OOH2012) to the SOC2002 (used in NLSY97). Next, we identify the entry education for each observed SOC2002 code. If a single SOC2002 occupation code corresponds to multiple SOC2010 codes, we compute the average education level for those SOC2010 occupations, and then take the education level that is closest to the computed average education level as its typical entry education.[27] Last, we manually adjust the education requirements for four SOC2002 occupations as most of their corresponding SOC2010 occupations require at least a bachelors degree, even if their average entry education is below a bachelors degree.[28] After determining the typical entry education for each SOC2002 occupation in our NLSY97 sample, we construct a binary variable to indicate whether an occupation is a college job or not.

Compared with the ONET50.00 identification, among the 298 occupations in the sample, 279(93.6%) are consistently identified using the OOH2012 definition. To be specific, 99 out of 108 (91.67%) ONET50.00 college occupations are also considered as college occupations by OOH2012 definition. Additionally, 180 out of 190 (94.74%) ONET50.00 non-college occupations remain classified as non-college occupations by OOH2012 definition. Meanwhile, 197,888 out of 214,029 (92.5%) of employment observations remain unaffected when adopting this alternative definition.[29]

[26]The OOH2012 lists eight entry education levels: (i) Less than high school; (ii) Postsecondary non-degree award; (iii) High school diploma or equivalent; (iv) Some college, no degree; (v) associates degree; (vi) bachelors degree; (vii) masters degree; and (viii) doctoral or professional degree.

[27]Take the SOC2002 job "Claims Adjusters, Appraisers, Examiners, and Investigators" as an example. It corresponds to two distinct SOC2010 occupations: Insurance Appraisers, Auto Damage and Claims Adjusters, Examiners, and Investigators. The entry educations are postsecondary non-degree award and high school diploma or equivalent separately. The average education level would be 2.5, so its entry education level would be high school diploma or equivalent.

[28]These four SOC2002 occupations are (i) Other Teachers and Instructors, (ii) Designers, (iii) Miscellaneous Community and Social Service Specialists, and (iv) Emergency Management Specialists.

[29]Here are some more details on the misalignment between the ONET50.00 and OOH2012 definitions in identifying college jobs where 19 (or 6.4%) occupations have different classifications. In particular, the OOH2012 classifies 10 of these occupations as college jobs, in contrast to their non-college job classification under the ONET50.00, a discrepancy we refer to as a Type 1 Misalignment. On the other hand, 9 occupations are identified as non-college jobs by the OOH2012 but as college jobs by the ONET50.00, a Type 2 Misalignment. Regarding the impact on employment observations in our sample, approximately 7.5% (16,141/214,029) are affected by these differences. This includes 6,014 observations (2.8%) affected by a Type 1 Misalignment and 10,127 observations (4.7%) impacted by a Type 2 Misalignment.

B.2 Empirical Evidence

In this section, we compare the empirical analysis presented in the main text for each definition of college jobs. Overall, these comparisons show that the nature of underemployment, especially the negative duration dependence, is not overly sensitive to the definition of a college job.

B.2.1 The Prevalence and Persistence of Underemployment

To examine whether the underemployment remains prevalent and persistent across different definitions, we replicate the exercise in the main text and look into the fraction of respondent's history spent in each labor force status. Table B.1 shows that the average underemployment ratio and underemployment durations are similar across the three definitions.

Table B.1: Comparisons – Labor Force Statuses

	ONET50.00	ONET42.27	OOH2012
Ratio			
Unemployed	0.031	0.031	0.031
Underemployed	0.392	0.367	0.408
Properly Employed	0.522	0.547	0.505
Duration (months)			
Unemployed	2.39	2.39	2.39
Underemployed	18.22	17.62	18.70
Properly Employed	22.62	22.71	22.36

B.2.2 Duration Dependence

To investigate how the magnitude of duration dependence reacts to different definitions, we re-estimate the duration dependence for each alternative definition using equation (2) and compare these estimates to the duration dependence identified under ONET50.00.

Figure B.2 displays the comparison across various definitions. Notably, the magnitude of duration dependence in each definition is similar, indicating that the principal characteristic of underemployment we are examining is not sensitive to the chosen identification method.

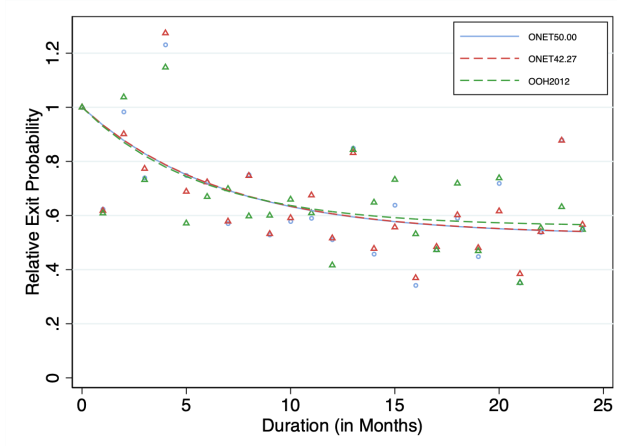


Figure B.2: Comparisons – Duration Dependence in Underemployment

B.2.3 Wages and Underemployment

To compare wage losses in college jobs across different definitions, we re-estimated the coefficients in equation (3) and present them in Table B.2. Specifically, we found that wage loss in college jobs becomes more severe under the alternative definitions. Given that the magnitude of this wage loss is informative of skill evolution parameters, a larger wage loss indicates a greater depreciation of college skills during underemployment, which potentially enhances the role of human capital changes in determining the duration dependence in underemployment. Consequently, we proceed to re-calibrate the model by targeting these new data moments. Specifically, we aim to investigate whether, with an increased skill loss during underemployment, unobserved heterogeneity still predominantly explains the duration dependence in underemployment.

Table B.2: Comparisons – Wages and Underemployment

	ONET50.00	ONET42.27	OOH2012
$\partial \log(w_n)/\partial v$	-0.0136	-0.0118	-0.0135
$\partial \log(w_c)/\partial v$	-0.0136	-0.0182	-0.0162
$\partial \log(w_n)/\partial \tau$	0.0006	0.0006	0.0006
$\partial \log(w_c)/\partial \tau$	-0.0013	-0.0019	-0.0016

B.3 Quantitative Analysis

In this section, we replicate the quantitative analysis for the alternative definitions of college jobs.

B.3.1 Calibration

We start with re-calibrating of the model by targeting the data moments computed under each alternative definition. These calibrations follow the same strategy as stated in Section 5.1. Regardless of the definition used, the model can match the targeted moments well. The comparisons of the transition path observed in the data with the model-generated transition path are depicted in Figures B.3(a) and B.4(a) while Table B.3 details the model fits for other targeted moments. Last, the calibrated parameters by each definition are listed in Table B.4.

Table B.3: Model Fits - Other Moments

	ONET50.00		ONET42.27		OOH2012	
	Target	Model	Target	Model	Target	Model
Unemployment rate	0.081	0.081	0.081	0.081	0.081	0.081
Underemployment rate	0.416	0.414	0.388	0.388	0.433	0.432
b/[labor productivity]	0.710	0.707	0.710	0.709	0.710	0.709
U2N duration	2.147	2.111	2.147	2.126	2.111	2.083
College job premium	0.260	0.259	0.243	0.246	0.275	0.280
$\partial \log(w_n)/\partial v$	-0.014	-0.014	-0.012	-0.012	-0.014	-0.014
$\partial \log(w_c)/\partial v$	-0.014	-0.014	-0.018	-0.018	-0.016	-0.016
$\partial \log(w_n)/\partial \tau$	0.001	0.001	0.001	0.001	0.001	0.001
$\partial \log(w_c)/\partial \tau$	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002

B.3.2 Decomposing Duration Dependence

With the calibrated models in hand, we can move on to explore the contribution of unobserved heterogeneity versus human capital dynamics to duration dependence in underemployment. To do so, we compute the duration dependence after turning off the channel of the human capital dynamics. Figure B.3 displays the decomposition outcome for the ONET42.27 definition. Notably, the model with only unobserved heterogeneity accounts for at least 91.5% of this decline.

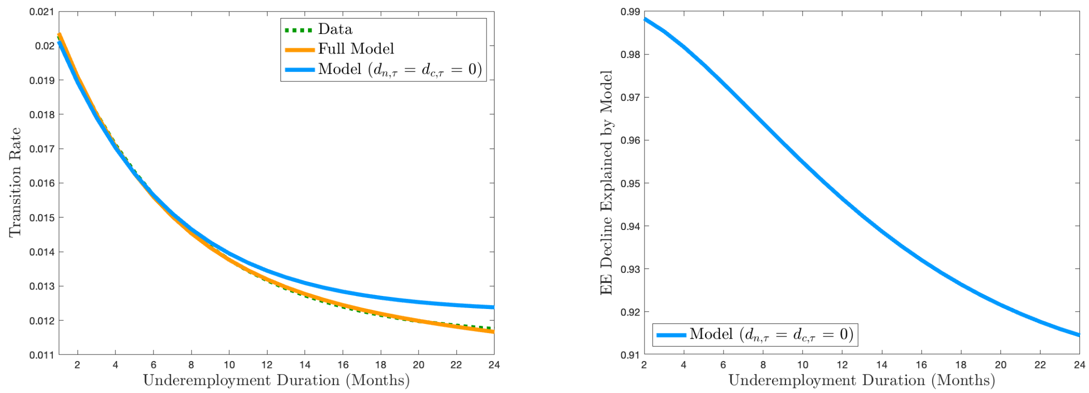
Table B.4: Calibrated Parameters

Parameter	Definition	ONET50.00	ONET42.27	OOH2012
β	Discount factor	0.996	0.996	0.996
δ	Entry/exit probability	0.011	0.011	0.012
g_c	College productivity	1.000	1.000	1.000
g_n	Non-college productivity	0.745	0.743	0.725
b	Utility while unemployed	0.611	0.612	0.601
k_n	Non-college vacancy cost	2.167	2.414	1.954
k_c	College vacancy cost	2.054	1.831	2.069
λ	Employed search intensity	0.851	0.856	0.852
a^L	Suitability pr.: type L	0.023	0.025	0.024
a^H	Suitability pr.: type H	0.354	0.354	0.405
π	Pr. of being a type H worker	0.049	0.051	0.037
ϕ	Pr. of regaining college skills	0.006	0.006	0.006
$d_{c,v}$	College skill loss: unemp.	-0.014	-0.019	-0.017
$d_{c,\tau}$	College skill loss: underemp.	-0.001	-0.002	-0.002
$d_{n,v}$	Non-college skill loss: unemp.	-0.014	-0.012	-0.014
$d_{n,\tau}$	Growth of non-college skills	0.001	0.001	0.001

To arrive at the aggregate decomposition, we calculate the weighted average of the fraction explained by unobserved heterogeneity across all durations τ. The weights are determined by the proportion of underemployed workers in steady-state at each duration τ. The model without human capital dynamics explains 93.0% of the duration dependence. Similarly, Figure B.4 presents the decomposition results for the OOH2012 definition. In this case, the model with only unobserved heterogeneity explains at least 92.5% of the decline. On the aggregate, the model without human capital dynamics accounts for 94.0% of the duration dependence.

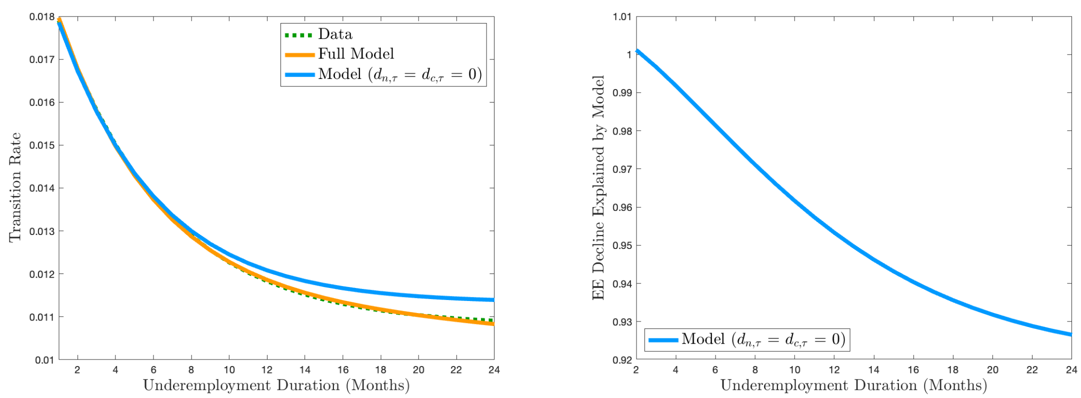
Additionally, we examine the extent to which information frictions contribute to the generation of underemployment and its duration dependence by analyzing the model with full information that still accounts for heterogeneity in worker suitability and shocks to occupation-specific human capital. Figures B.5-B.6 illustrate the duration dependence with and without information frictions for the alternative definitions. The results in both definitions mirror the patterns observed in the baseline exercise.

First, in the model of full information where workers' types are publicly known, work-



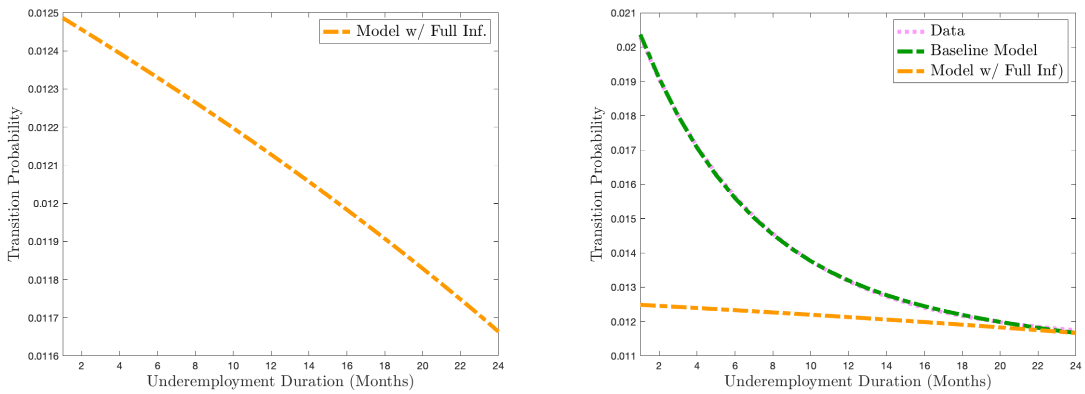
(a) Transition Probability Path (b) Fraction Explained at each τ

Figure B.3: ONET42.27 Duration Dependence Decomposition



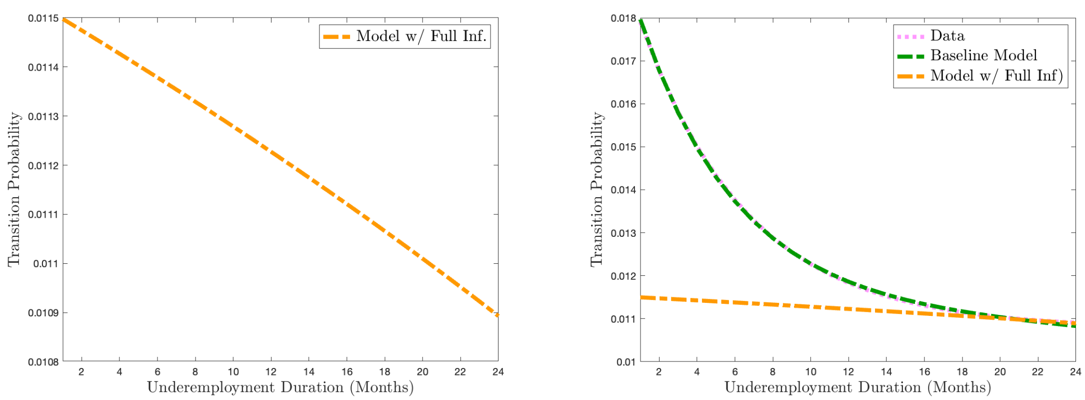
(a) Transition Probability Path (b) Fraction Explained at each τ

Figure B.4: OOH2012 Duration Dependence Decomposition ers with broad suitability do not search for non-college jobs. Second, a mild duration dependence is still noticeable among workers with limited suitability who seek non-college jobs and become stuck in them. However, the magnitude of duration dependence becomes much smaller compared to what is observed in both the full model and the data. These observations collectively indicate that the information friction is crucial in generating the underemployment and the observed duration dependence in underemployment.



(a) No Information Frictions (b) Model and Data Comparisons

Figure B.5: ONET42.27 Duration Dependence with and without Information Frictions

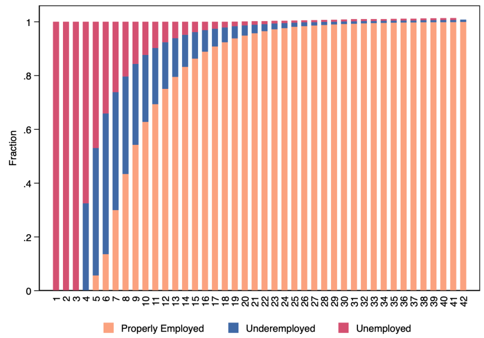


(a) No Information Frictions (b) Model and Data Comparisons

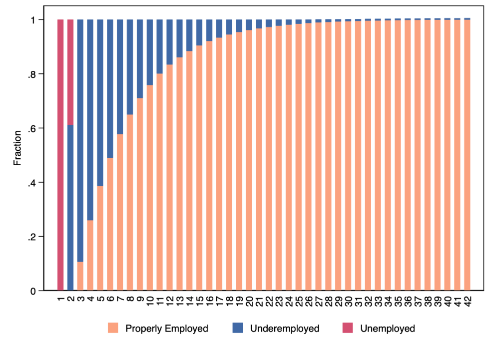
Figure B.6: OOH2012 Duration Dependence with and without Information Frictions

B.3.3 Sorting and Bad Luck

So far, our comparisons indicate that unobserved heterogeneity among college graduates accounts for the majority of the duration dependence observed in underemployment.

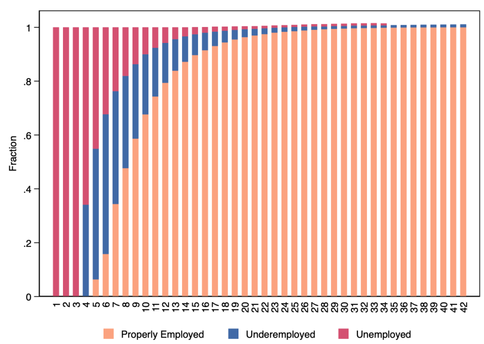


(a) Unlucky Workers

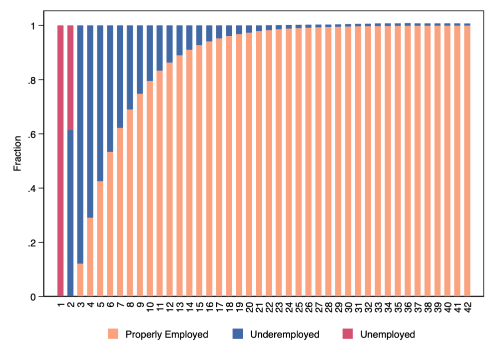


(b) Lucky Workers

Figure B.7: ONET42.27 Percentage of Time Spent in Various Labor Market Statuses



(a) Unlucky Workers



(b) Lucky Workers

Figure B.8: OOH2012 Percentage of Time Spent in Various Labor Market Statuses

This section replicates the exercise presented in the Section 5.3, which attempts to disentangle between sorting and bad luck in generating long underemployment spells.

Figure B.7 compares the fraction of each month spent in each labor market status between luck and unlucky workers under the ONET42.27 definition. The average duration of underemployment for the unlucky group is 5.41 months, compared to 5.36 months for the lucky group. Similarly, Figure B.8 demonstrates the time allocation in each labor market status for the two groups under the OOH2012 definition, where the average underemployment duration for the unlucky and lucky groups is 4.88 months and 4.86 months, respectively.

Table B.5 presents the correlation between the duration of underemployment and its preceding unemployment duration. Notably, under each definition, the correlation between the two durations is very close to zero, just as we observe in the data.

Table B.5: Correlation between v and τ

	Data		Model
	Unconditional	Conditional	
ONET42.27 Definition			
First UN transitions	-0.005 (0.903)	0.016 (0.919)	-0.021
UN transitions	-0.026 (0.424)	0.046 (0.678)	
OOH2012 Definition			
First UN transitions	-0.027 (0.515)	0.060 (0.725)	-0.024
UN transitions	-0.037 (0.234)	0.034 (0.747)	

Notes: P-values in parentheses.

C Calibration Appendix

C.1 Data Moments for Calibration

C.1.1 Wage Premium

To obtain this target, we estimate the following regression:

$$w_{it} = \beta \text{College}_{it} + \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \tag{C.1}$$

where w_{it} is the log wage of individual i in time t, College is an indicator for whether individual i works in a college occupation, δ_i is an individual fixed effect, and X contains a cubic in potential experience, regional annual unemployment rate, aggregate annual unemployment rate, a quadratic in age, industry (2-digit), regional, month, and year fixed effects. We follow Barnichon and Zylberberg (2019) in estimating equation (C.1) on "marginally" underemployed workers only, i.e., workers who transitioned from proper employment to underemployment and back to proper employment to control for selection based on unobservable characteristics into underemployment.

Table C.1 contains the estimation of wage premium for college jobs. Each column represents a different combination of control variables and fixed effects. Column (4) represents our preferred specification that is used to calibrate the model in Section 5.1.

Table C.1: The Wage Premium of College Jobs

	(1)	(2)	(3)	(4)
College	0.3849***	0.3857***	0.2600***	0.2597***
	(0.0192)	(0.0196)	(0.0218)	(0.0223)
Exp	-0.0087***	-0.0026	0.0013	-0.0062***
	(0.0023)	(0.0020)	(0.0019)	(0.0021)
Exp^2	0.0002***	0.0000	-0.0000	0.0001**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	-0.0000	0.0000	0.0000**	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annual Urate				-0.0179**
				(0.0070)
Annual Urate				-0.0083
				(0.0064)
Age				0.4603***
				(0.0920)
Age^2				-0.0084***
				(0.0018)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,085	10,988	10,988	10,988
R^2	0.843	0.853	0.894	0.894

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of College. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

Table C.2 contains the estimation of wage premium by estimating equation (C.1) for college jobs identified by the ONET42.27 definition. Column (4) represents our preferred specification that is used to calibrate the ONET42.27 model in Section B.3.1.

Table C.2: The Wage Premium of College Jobs (ONET42.27 Definition)

	(1)	(2)	(3)	(4)
College	0.3382***	0.3390***	0.2409***	0.2426***
	(0.0184)	(0.0189)	(0.0228)	(0.0230)
Exp	-0.0081***	-0.0017	0.0010	-0.0102***
	(0.0025)	(0.0021)	(0.0020)	(0.0022)
Exp^2	0.0002*	-0.0000	-0.0001*	0.0001**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	-0.0000	0.0000*	0.0000***	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annaul Urate				-0.0110
				(0.0070)
Annaul Urate				-0.0070
				(0.0067)
Age				0.8568***
				(0.1308)
Age^2				-0.0159***
				(0.0026)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,209	11,112	11,112	11,112
R^2	0.823	0.833	0.883	0.884

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of College. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

Table C.3 contains the estimation of wage premium by estimating equation (C.1) for college jobs identified by the OOH2012 definition. Column (4) represents our preferred specification that is used to calibrate the OOH2012 model in Section B.3.1.

Table C.3: The Wage Premium of College Jobs (OOH2012 Definition)

	(1)	(2)	(3)	(4)
College	0.4182***	0.4076***	0.2766***	0.2747***
	(0.0207)	(0.0207)	(0.0208)	(0.0212)
Exp	0.0007	0.0078***	0.0108***	0.0033
	(0.0023)	(0.0020)	(0.0021)	(0.0022)
Exp^2	-0.0000	-0.0002***	-0.0002***	-0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	0.0000	0.0000***	0.0000***	0.0000***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annaul Urate				-0.0115*
				(0.0065)
Annaul Urate				0.0002
				(0.0058)
Age				0.5330***
				(0.1016)
Age^2				-0.0096***
				(0.0020)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,096	10,993	10,993	10,993
R^2	0.845	0.854	0.896	0.896

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of College. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

C.1.2 Unemployment and Underemployment Rates

The calculation of the unemployment and underemployment rates involves a three-step procedure. First, the number of individuals in the categories of unemployment, underemployment, and proper employment are counted in each week. Second, the proportion of each labor status category is calculated by dividing the headcount of each category by the total headcount observed during that week. Finally, both an unweighted average and a weighted average are computed across all weeks, with the number of observations in that week serving as the weighting factor. Table C.4 contains the results for each definition of college jobs.

Table C.4: Composition of Labor Force Statuses

	Unemployed	Underemployed	Properly employed	NILF
ONET50.00				
Unweighted	0.032	0.408	0.497	0.063
Weighted	0.027	0.416	0.503	0.054
ONET42.27				
Unweighted	0.032	0.385	0.520	0.063
Weighted	0.027	0.388	0.531	0.054
OOH2012				
Unweighted	0.032	0.435	0.470	0.063
Weighted	0.027	0.433	0.485	0.054

C.1.3 Average U2N Duration

To determine the average U2N duration, we compute the time it takes for the new entrant to secure their first non-college job. To keep consistency with our model, we presume that each college graduate initially has a 4-week (or 1-month) period of unemployment. We then track the number of months each unemployed graduate spends before finding their first non-college job. Next, this duration is averaged across individuals who have ever experienced underemployment. Ultimately, we find the average U2N duration to be 2.147/2.147/2.111 months under the ONET50.00/ONET42.27/OOH2012 definition for college jobs.

C.2 Calibration with Optimal Weighting Matrix

C.2.1 Methodology

We use the Method of Simulated Moments (MSM) to estimate parameters ($\hat{\vartheta}$) by minimizing the weighted distance between empirical moments (m) and simulated moments ($\tilde{m}$). In the baseline calibration, we use a scaled identity matrix, I/m^2, to compute the weighted distance. This approach minimizes the fractional deviation of the simulated moment from its corresponding data moment by normalizing the scales or units of the data moments, which ensures that no single moment disproportionately impacts the estimation due to its scale. The efficiency of MSM estimates can be improved by choosing a more efficient weighting matrix W^* to minimize the variance of the MSM estimator. In particular, a more efficient weighting matrix W^* is the inverse variance-covariance matrix of data moments, denoted as $W^* = S_m^{-1}$, that puts less weight on moments with greater variance.

To construct the optimal weighting matrix, we start by determining the variance-covariance matrix of data moments (S_m). This is achieved by generating bootstrap samples and computing the data moments in each sample. Specifically, by randomly selecting observations from our full sample with replacement, we create a bootstrap sample identical in size to the original dataset. This step is repeated 1000 times, and each iteration yields a new bootstrap sample. Within each of these samples, we compute the moments targeted in the calibration. After collecting all data moments computed in each bootstrap sample, we compute the variance-covariance matrix (S_m). In this matrix, the diagonal elements denote the variances of each data moment, while the off-diagonal elements reflect the covariances between different moments.

Finally, we re-estimate the unknown parameters ($\hat{\vartheta}^*$) by minimizing the weighted deviation of the model moments from their corresponding data moments, where the weighting matrix is given by the inverse variance-covariance matrix. That is,

$$\hat{\vartheta}^* = \text{argmin } (\tilde{m} - m)' W^* (\tilde{m} - m), \ W^* = S_m^{-1}. \tag{C.2}$$

C.2.2 Comparison with the Baseline Calibration

The model fit using the optimal weighting matrix, and its comparison with the baseline calibration, are depicted in Figure C.1 and Table C.5. It is evident that the optimal weighting matrix does not significantly improve the model fit compared to the baseline calibration. The parameters calibrated using the optimal weighting matrix, and their comparison with those from the baseline calibration, are detailed in Table C.6. Notably, the parameters calibrated under the optimal weighting matrix are not very different from those in the baseline calibration. Consequently, we adopt the model calibrated with the scaled identity matrix for the quantitative analysis in the main text.

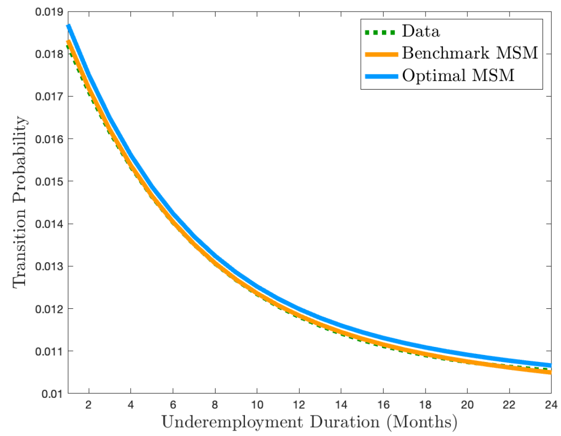


Figure C.1: Comparison between Baseline and Optimal MSM - Transition Path

Table C.5: Comparison between Baseline and Optimal MSM - Other Moments

Moment	Target	$\tilde{m}(\vartheta)$	$\tilde{m}(\vartheta^*)$
Unemployment rate	0.081	0.081	0.081
Underemployment rate	0.416	0.414	0.411
U2N duration	2.147	2.111	2.110
College job premium	0.260	0.259	0.260
b/[Average labor productivity]	0.710	0.707	0.707
$\partial \log(w_n)/\partial v$	-0.014	-0.014	-0.014
$\partial \log(w_c)/\partial v$	-0.014	-0.014	-0.014
$\partial \log(w_n)/\partial \tau$	0.001	0.001	0.001
$\partial \log(w_c)/\partial \tau$	-0.001	-0.001	-0.001

C.2.3 Over-identification Test

Given that the number of data moments ($p = 33$) is greater than the number of unknown parameters ($d = 14$), the model is over-identified. When the model is over-identified, some moment conditions will be different from zero, which allows us to assess how well the estimated model matches the data. Following Jalali et al. (2015), we quantify the significance of calibration error between the model moments ($\tilde{m}$) and data moments (m) by computing the following J-statistic:

$$J = (\tilde{m} - m)' W^* (\tilde{m} - m) \sim \chi^2_{p-d}, \; W^* = S_m^{-1}. \tag{C.3}$$

Table C.6: Comparison between Baseline and Optimal MSM - Parameter Values

	Definition	$\hat{\vartheta}$	$\hat{\vartheta}^*$
β	Discount factor	0.996	0.996
δ	Entry/exit probability	0.011	0.011
g_c	College productivity	1.000	1.000
g_n	Non-college productivity	0.745	0.744
b	Utility while unemployed	0.611	0.611
k_n	Non-college vacancy cost	2.167	2.173
k_c	College vacancy cost	2.054	2.043
λ	Employed search intensity	0.851	0.857
a^L	Suitability pr.: type L	0.023	0.023
a^H	Suitability pr.: type H	0.354	0.356
π	Pr. of being a type H worker	0.049	0.049
ϕ	Pr. of regaining college skills	0.006	0.006
$d_{c,v}$	College skill loss: unemp.	-0.014	-0.014
$d_{c,\tau}$	College skill loss: underemp.	-0.001	-0.001
$d_{n,v}$	Non-college skill loss: unemp.	-0.014	-0.014
$d_{n,\tau}$	Growth of non-college skills	0.001	0.001

For the calibration with $W^* = S_m^{-1}$, the J-statistic stands at $1.673 < \chi^2_{p-d} = 36.191$, indicating that at a 99% confidence level, there is no statistical difference between the estimated model and the true data-generating process.

C.3 Extended Model with Output Difference across Suitability Types

C.3.1 Model

To support our identifying assumption, we extend the model to incorporate the output in college jobs as a function of the worker's suitability. Specifically, we assume:

$$y_c^H(v,\tau) = \alpha y_c^L(v,\tau). \tag{C.4}$$

As the only modification to the model is the production technology in college jobs, we use this section to present the surplus of a match between a worker and a college job.

Consider a worker with history (v,τ) and expected suitability μ who forms a new match with a college job. The expected output of the match is $\mu y_c^H(v,\tau) + (1-\mu) y_c^L(v,\tau)$.

After producing for one period, the worker and firm can learn the worker's suitability type, as the match either produces $y_c^H(v,\tau)$ or $y_c^L(v,\tau)$ units of output. In the former (latter) case, the worker and firm learn that the worker has broad (limited) suitability. Let $V_{e,c}^i(v,\tau)$ represent the total surplus of a match between a worker with history (v,τ) and is known to be type-i. It follows that

$$V_{e,c}^i(v,\tau) = y_c^i(v,\tau) + \beta(1-\delta)\{\phi V_{e,c}^i(1,0) + (1-\phi)V_{e,c}^i(v,\tau)\}. \tag{C.5}$$

Equation (C.5) has the same interpretation as equation (8) in the main text, except that the match output is indexed by the worker's suitability type.

Now let $V_{e,c}(v,\tau,\mu)$ denote the surplus at the formation of a match between a college job and worker with history (v,τ) and expected suitability μ. It follows that $V_{e,c}(v,\tau,\mu)$ satisfies

$$\begin{aligned} V_{e,c}(v,\tau,\mu) = \mu y_c^H(v,\tau) + (1-\mu)y_c^L(v,\tau) + \beta(1-\delta)\big\{\mu[\phi V_{e,c}^H(1,0) + (1-\phi)V_{e,c}^H(v,\tau)] \\ + (1-\mu)[\phi V_{e,c}^L(1,0) + (1-\phi)V_{e,c}^L(v,\tau)]\big\}, \end{aligned} \tag{C.6}$$

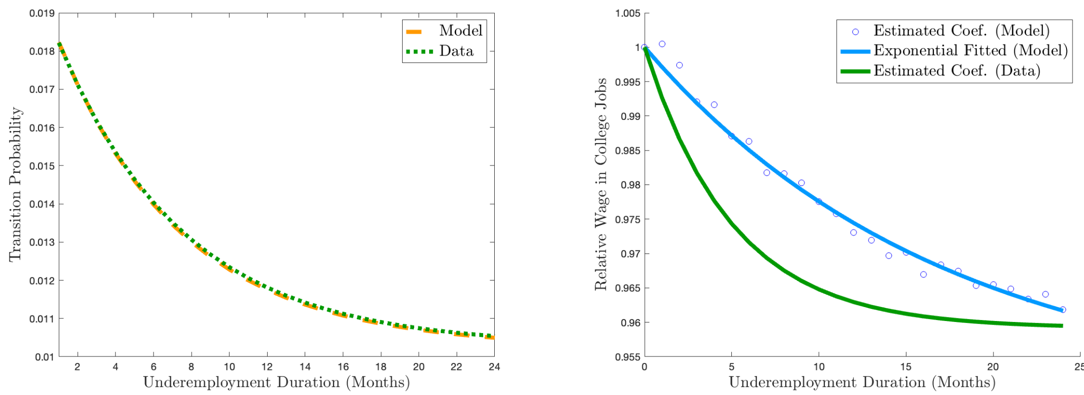
where $V_{e,c}^i(v,\tau)$ satisfies (C.5). From (C.6), a new match produces the expected output $\mu y_c^H(v,\tau) + (1-\mu)y_c^L(v,\tau)$. With probability μ, the worker is a broad-suitable worker and therefore the match surplus in subsequent periods is determined by $V_{e,c}^H(v,\tau)$. With probability $1-\mu$, the worker has a limited suitability and the match surplus in future periods is given by $V_{e,c}^L(v,\tau)$. Notice throughout equations (C.5) and (C.6) that we still account for the possibility of workers regaining their college skills.

With $V_{e,c}(v,\tau,\mu)$ in hand, one can compute the entry of firms into submarkets with college jobs using the entry condition, equation (9). The rest of the model's equilibrium conditions are unchanged relative to Section 4.1.

C.3.2 Calibration and Decomposition

This calibration approach is similar to the baseline model. However, instead of targeting the estimated wage effects from column (6) of Table 2, we target the relative wages in college jobs as a function of underemployment history that's generated by estimating the negative exponential model on college wages shown in Figure A.7(a). This is informative for calibrating α in equation (C.4), as the wage pattern, especially the degree of convexity in the wage decline in college jobs, is informative of the role of unobserved heterogeneity in determining output and wages in college jobs.

In a nutshell, we calibrate 15 parameters by targeting the transition path (24 moments), the path of college job wages (25 moments), as well as other 5 moments from the baseline calibration. Figure C.2 and Table C.7 present the fit of the extended model. The calibrated parameters are listed in Table C.8.



(a) Transition Path (b) College Job Wage Path

Figure C.2: Model Fit

Table C.7: Model and Data Comparison

Moment	Target	Model
Unemployment rate	0.081	0.083
Underemployment rate	0.416	0.415
U2N duration	2.147	2.118
College job premium	0.260	0.273
b/[Average labor productivity]	0.710	0.707

We proceed to the decomposition exercise presented in Section 5.2. Figure C.3 shows that, by deactivating skill accumulation and loss during underemployment, the transition probability for each underemployment history τ increases slightly. On the aggregate, the fraction explained by the model augmented with output differences across suitability types, becomes slightly larger as it introduces an additional channel through which selection contributes to the duration dependence of underemployment. Specifically, the model with only unobserved heterogeneity can explain 98.80% of the decline in transition probability observed in the data, which is slightly higher than what could be explained by the unobserved heterogeneity in the baseline model (95.27%).

Table C.8: Parameter Values

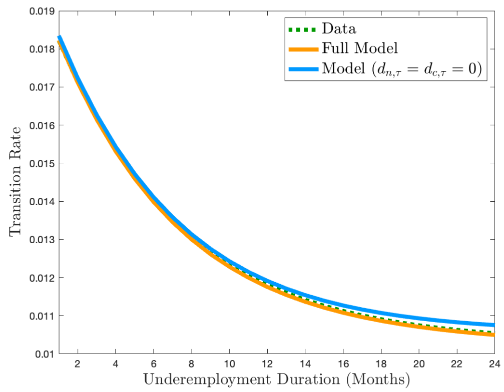
	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.354
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.051
g_n	Non-college productivity	0.750	ϕ	Pr. of regaining college skills	0.006
b	Utility while unemployed	0.617	$d_{c,v}$	College skill loss: unemp.	-0.0142
k_n	Non-college vacancy cost	2.255	$d_{c,\tau}$	College skill loss: underemp.	-0.0004
k_c	College vacancy cost	2.046	$d_{n,v}$	Non-college skill loss: unemp.	-0.0138
λ	Employed search intensity	0.833	$d_{n,\tau}$	Growth of non-college skills	0.0006
α	Prod. of type-H workers	1.070	-	-	-

C.4 Robustness of Duration Dependence Decomposition

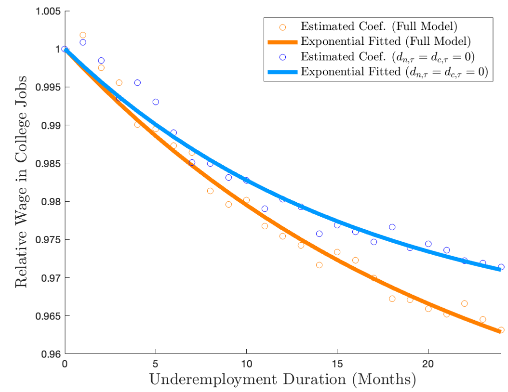
C.4.1 With Pre-set Skill Parameters

Instead of calibrating the skill accumulation and loss parameters to match the wage effects in column (6) of Table 2, we can instead rely on previous literature which evaluates the effect of nonemployment on measures of productivity and to set the skill parameters equal to what that literature has found. In other words, we can simply set the skill parameters so that they are in line with literature that evaluates the effect of nonemployment on productivity. To do this, we draw on the recent study by Dinerstein et al. (2022) who exploited quasi-random variation in teaching assignments in Greece to estimate the rate of skill depreciation and the returns to experience. While the setting is specific to teachers in Greece, this study provides what is arguably the best evidence to date on the effect of nonemployment and working on productivity at the individual level. They find a skill depreciation rate of 4.3% per year and a returns to experience of 6.8%. Therefore, the net effect of working on productivity is $6.8 - 4.3 = 2.5\%$ per year. With these estimates in mind, we set $d_{c,\tau} = -(.043)^{\frac{1}{12}} - 1 = -0.0035$ and $d_{n,\tau} = (0.025)^{\frac{1}{12}} - 1 = .0021$, which are simply monthly rates that correspond with the annual rates estimated by Dinerstein et al. (2022).[30] For the probability of college workers regaining their skills, we set ϕ so that the average increase in productivity after working in college jobs is 0.21% per month. Given that the magnitude of dynamics for non-college skills (0.0021) and college skills

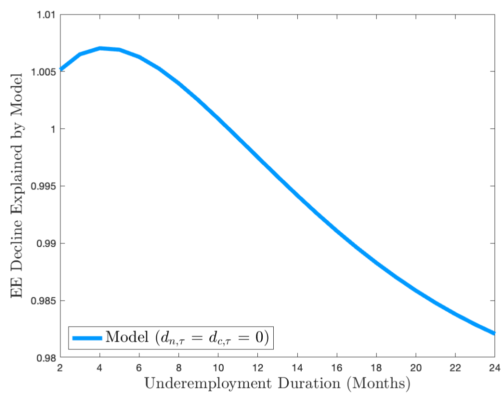
[30]Note that skill losses during unemployment are maintained at the same magnitude as in the baseline calibration.



(a) Transition Path



(b) Wage Path



(c) Fraction Explained at each τ

Figure C.3: Duration Dependence Decomposition

(-0.0035) during underemployment is nearly three to four times that of the estimated wage loss in the main text, which are 0.0006 and -0.0013 respectively, this exercise gives skill dynamics an opportunity to explain a larger proportion of the duration dependence in underemployment.

With the pre-set skill parameters in hand, we re-calibrate the model to target the transition path and the growth of college-job wages, as well as the moments listed in Table 3, except for the four wage effect targets. Figure C.4 and Table C.9 present the fit of the model with pre-set skill parameters. Notably, the re-calibrated model fits the data well. The calibrated parameters are listed in Table C.10.

We also replicate the decomposition exercise presented in Section 5.2. Figure C.5 shows that by turning off skill accumulation and loss during underemployment, the transition probability at each underemployment history τ increases by a small amount. Remarkably, the model with unobserved heterogeneity only explains 95.0% of the duration dependence in underemployment, which is less than the explanation provided by the full model. This discrepancy is due to the pre-set skill accumulation and decay rates in the full model being larger than those in the baseline model, thereby enhancing the model's ability to account for duration dependence through skill dynamics more effectively.

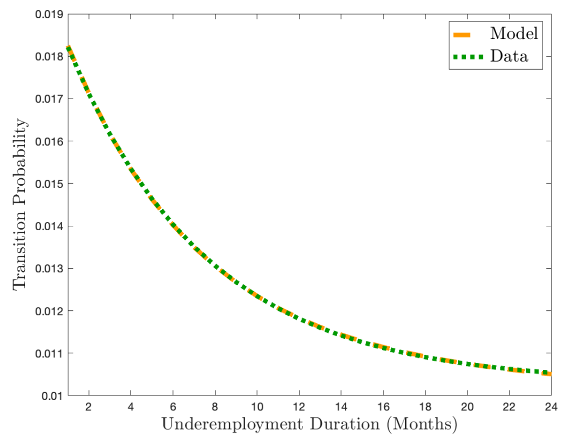


Figure C.4: Model Fit – Transition Path

Table C.9: Model and Data Comparison

Moment	Target	Model
Unemployment rate	0.081	0.081
Underemployment rate	0.416	0.416
U2N duration	2.147	2.105
College job premium	0.260	0.261
b/[Average labor productivity]	0.710	0.703
Recovery rate	0.0021	0.0021

C.4.2 Effective Wages

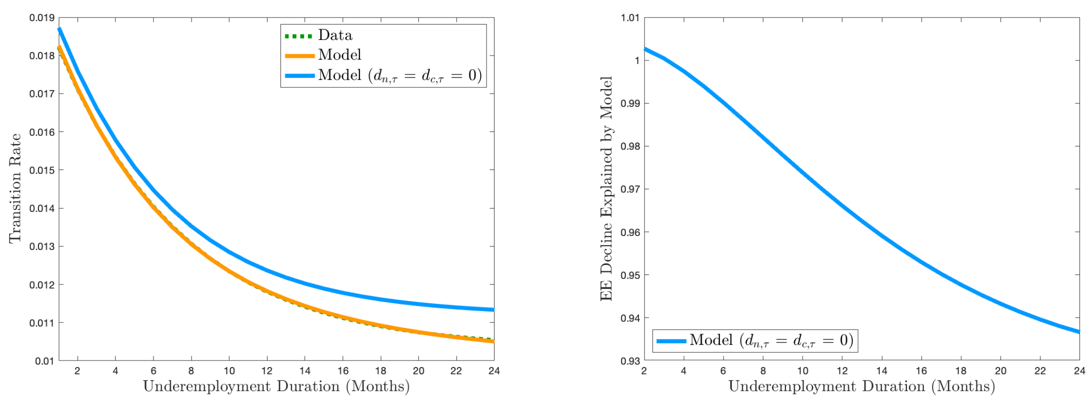
Given the free entry condition, equation (9), the value of the worker's employment contract can be expressed as

$$x(\chi, v, \tau, \theta) = V_{e,\chi}(v, \tau) - \frac{k_\chi}{q(\theta_{\chi,v,\tau})}. \tag{C.7}$$

Recall that a worker's wage is equated with the output of the match. Therefore, the worker earns the entire value of the match, $V_{e,\chi}(v, \tau)$. It follows from (C.7) that the fee

Table C.10: Parameter Values

	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.350
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.049
g_n	Non-college productivity	0.727	ϕ	Pr. of regaining college skills	0.061
b	Utility while unemployed	0.617	$d_{c,v}$	College skill loss: unemp.	-0.0136
k_n	Non-college vacancy cost	1.605	$d_{c,\tau}$	College skill loss: underemp.	-0.0035
k_c	College vacancy cost	2.661	$d_{n,v}$	Non-college skill loss: unemp.	-0.0136
λ	Employed search intensity	0.915	$d_{n,\tau}$	Growth of non-college skills	0.0021



(a) Path of Transition Probabilities

(b) Fraction Explained at each τ

Figure C.5: Duration Dependence Decomposition

paid by the worker to the firm upon the formation of the match is given by $k_\chi/q(\theta)$, i.e. the average recruiting costs incurred by the firm. To derive the worker's effective wage (wage net of a per-period fee paid to the firm), we first need to derive an explicit expression for the per-period fee paid by the worker. Note that the present discounted value of the per-period fee payments should sum up the aggregate fee, $k_\chi/q(\theta)$. Let $\hat{\xi}_\chi(v,\tau)$ denote the per-period fee paid by the worker with characteristics (v,τ) at the beginning of the match to a type χ firm. It is straightforward to show

$$\hat{\xi}_\chi(v,\tau) = \begin{cases} \frac{1-\beta(1-\delta)}{q(\theta(n,v,\tau,x))}\kappa_c & \text{if } \chi = c, \\ \frac{1}{q(\theta((c,v,\tau,x)))\{1+\sum_{\tau=1}^{\bar{\tau}}[\beta(1-\delta)]^\tau \prod_{k=1}^{\tau}[1-\mu_k\lambda p(\theta(c,v,\tau,x)))]\}}\kappa_n & \text{if } \chi = n, \end{cases} \tag{C.8}$$

where the second line of (C.8), the fee paid to non-college firms, accounts for the chance that the worker transitions to proper employment. The effective wage is given by $\tilde{w}_\chi(v,\tau) = y_\chi(v,\tau) - \hat{\xi}_\chi(v,\tau)$.

Next, we re-calibrate the model with the same calibration strategy as outlined in Section 5.1, except we use the effective wages. Figure C.6 and Table C.11 show that the model aligns closely with the data, while Table C.12 presents the parameter values.

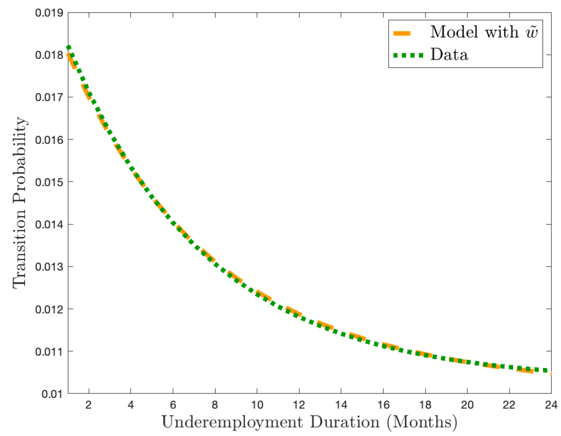


Figure C.6: Model Fit – Transition Path

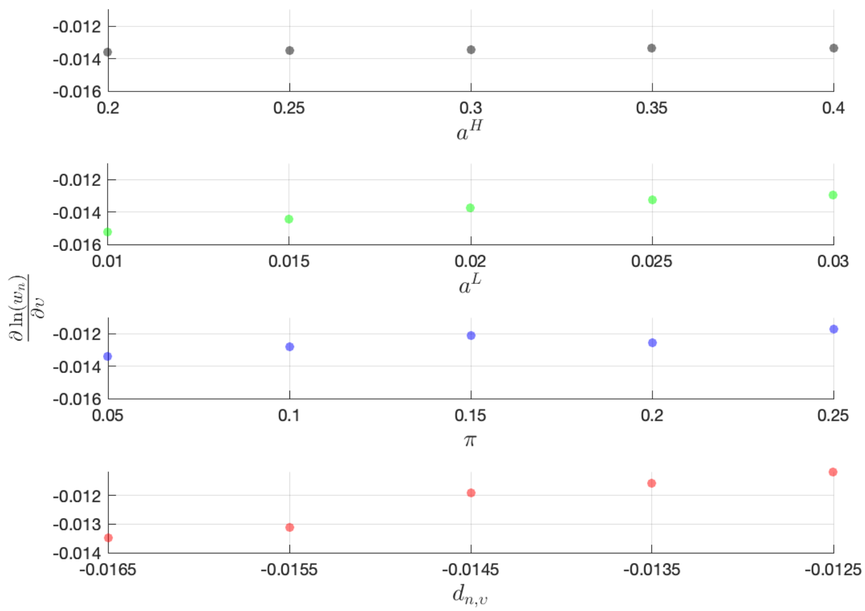
Table C.11: Model and Data Comparison

Moment	Target	Model	Moment	Target	Model
Unemployment rate	0.081	0.080	$\partial \log(w_n)/\partial v$	-0.014	-0.013
Underemployment rate	0.416	0.424	$\partial \log(w_c)/\partial v$	-0.014	-0.014
U2N duration	2.147	2.092	$\partial \log(w_n)/\partial \tau$	0.001	0.001
College job premium	0.260	0.266	$\partial \log(w_c)/\partial \tau$	-0.001	-0.001
b/[Average labor productivity]	0.710	0.766	-	-	-

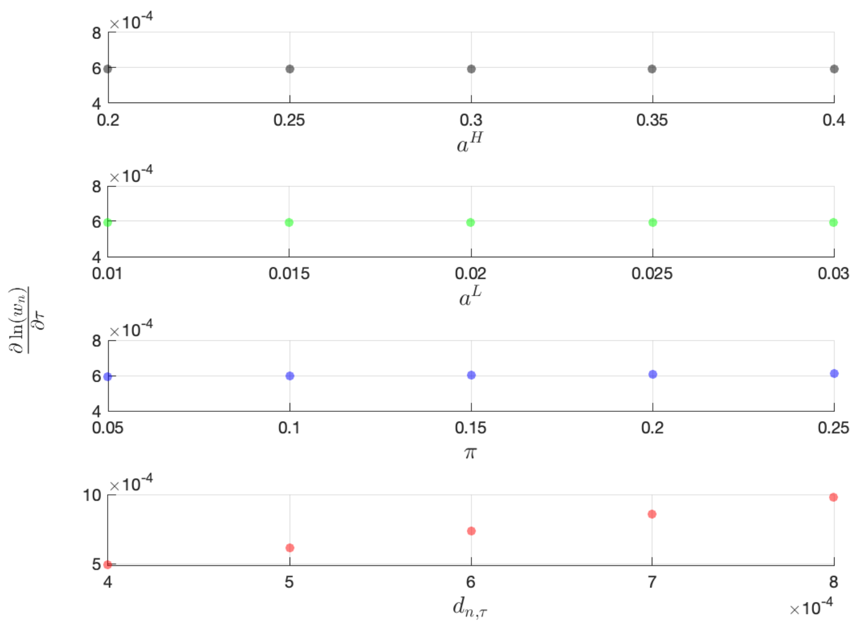
To support our identification strategy, Figure C.7 shows that the wage effects in non-college jobs are responsive to the skill loss and accumulation parameters $d_{n,v}$ and $d_{n,\tau}$ while not being responsive to the unobserved heterogeneity parameters. Figure C.8 shows the same, but for effective wages in college jobs. Moreover, Figure C.9 shows that the path of transition probabilities is responsive to changes in the unobserved heterogeneity parameters, while Figure C.10 demonstrates that changes in the skill loss and accumulation parameters have little effect on the transition path.

Table C.12: Parameter Values

	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.025
δ	Entry/exit probability	0.012	a^H	Suitability pr.: type H	0.355
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.052
g_n	Non-college productivity	0.900	ϕ	Pr. of regaining college skills	0.050
b	Utility while unemployed	0.724	$d_{c,v}$	College skill loss: unemp.	-0.012
k_n	Non-college vacancy cost	1.933	$d_{c,\tau}$	College skill loss: underemp.	-0.001
k_c	College vacancy cost	0.699	$d_{n,v}$	Non-college skill loss: unemp.	-0.016
λ	Employed search intensity	0.770	$d_{n,\tau}$	Growth of non-college skills	0.001

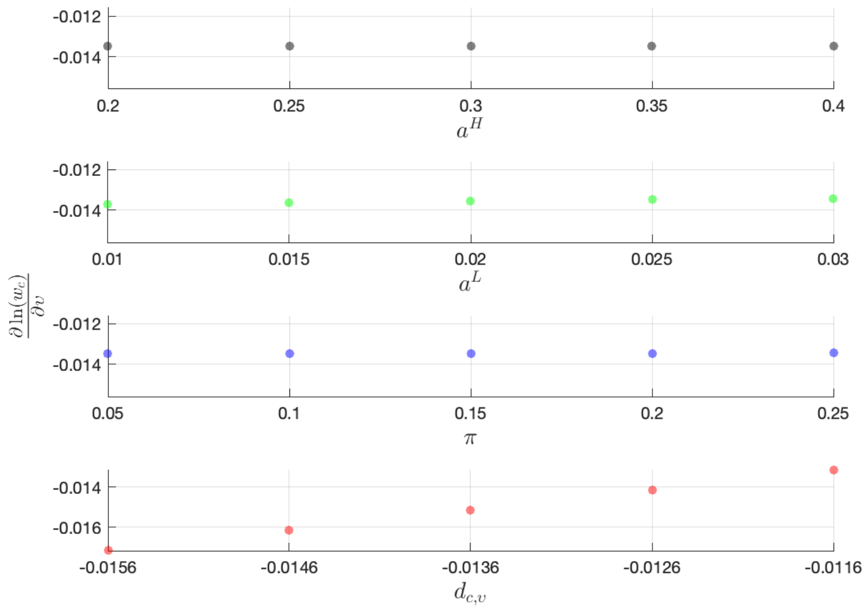


(a) $\partial \log(w_n)/\partial v$

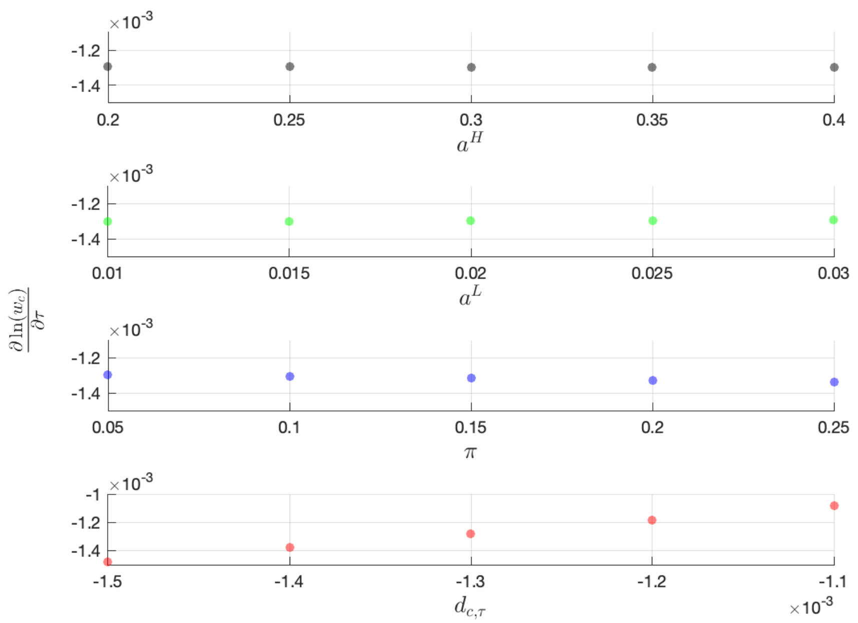


(b) $\partial \log(w_n)/\partial \tau$

Figure C.7: Comparative Statics of Wage Effects in Non-college Jobs

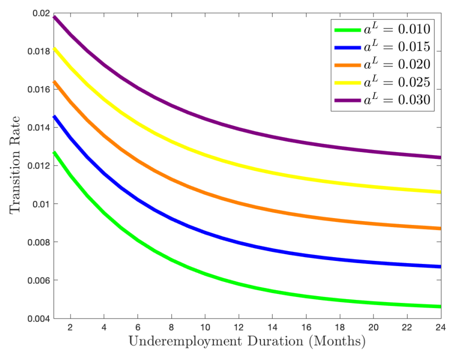


(a) $\partial \log(w_c)/\partial v$

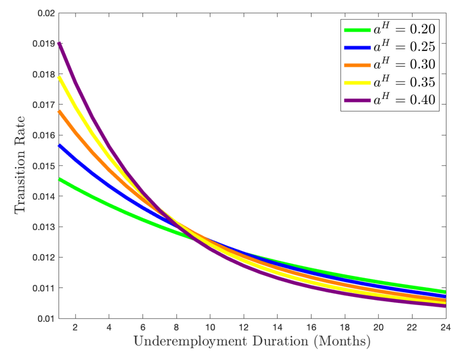


(b) $\partial \log(w_c)/\partial \tau$

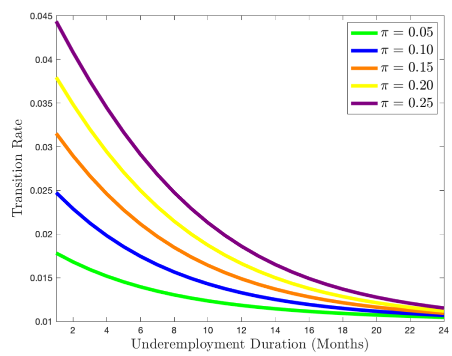
Figure C.8: Comparative Statics of Wage Effects in College Jobs



(a) Transition Path and a^L

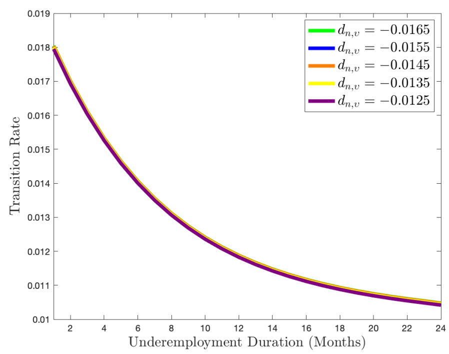


(b) Transition Path and a^H

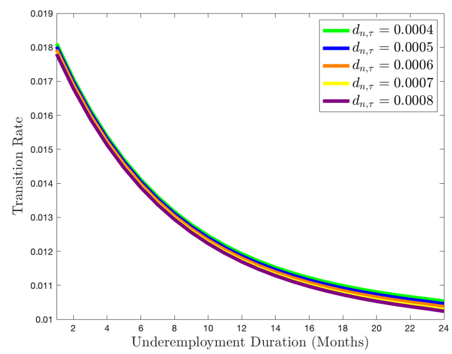


(c) Transition Path and π

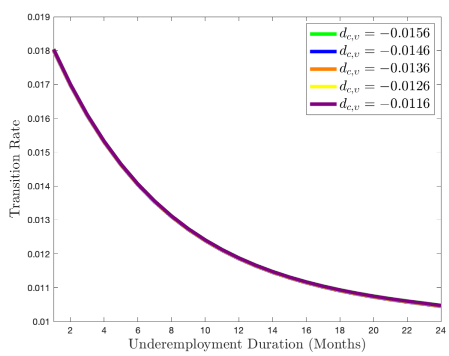
Figure C.9: The Transition Path and Unobserved Heterogeneity Parameters



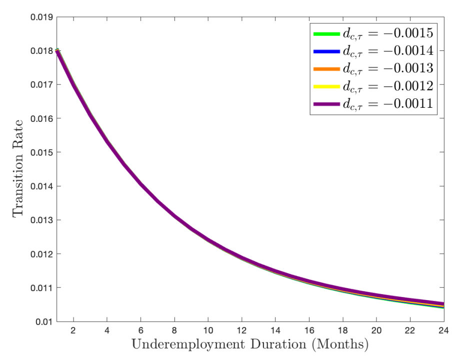
(a) Transition Path and $d_{n,v}$



(b) Transition Path and $d_{n,\tau}$



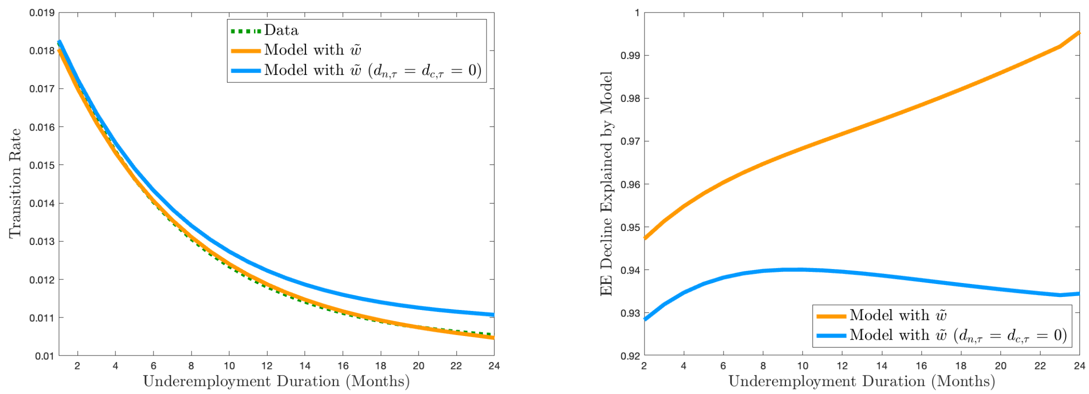
(c) Transition Path and $d_{c,v}$



(d) Transition Path and $d_{c,\tau}$

Figure C.10: The Transition Path and Skill Growth/Decay Parameters

Next, we turn off the accumulation and loss of skills during underemployment to assess how much of the duration dependence observed in the data can be explained by unobserved heterogeneity. The findings, as depicted in Figure C.11, reveal that shutting off skill dynamics during underemployment marginally alters the transition path. As for an aggregate decomposition, we find that 93.54% of the decline in transition probability is accounted for by unobserved heterogeneity.



(a) Path of Transition Probabilities

(b) Fraction Explained at each τ

Figure C.11: Duration Dependence Decomposition

D Theoretical Appendix

D.1 Laws of Motion

Let $u(v)$ denote the measure of workers who begin the period unemployed with unemployment history v. The law of motion for unemployed workers is given by

$$\hat{u}(v) = \begin{cases} (1-\delta)\delta & \text{for } v = 1, \\ (1-\delta)u(v_-)[\varrho_{n,v_-}(1-p(\theta^*_{n,v_-})) + \varrho_{c,v_-}(1-\mu_{v_-}p(\theta^*_{c,v_-}))] & \text{for } v \in \{2,\ldots,\bar{v}-1\}, \\ (1-\delta)\sum\limits_{v=\bar{v}}^{\bar{v}+1} u(v_-)[\varrho_{n,v_-}(1-p(\theta^*_{n,v_-})) + \varrho_{c,v_-}(1-\mu_{v_-}p(\theta^*_{c,v_-}))] & \text{for } v = \bar{v}, \end{cases} \tag{D.1}$$

where $\hat{u}(v)$ is the measure of unemployed workers with unemployment history v at the beginning of the next period, $v_- \equiv v-1$, $\varrho_{\chi,v} \in [0,1]$ is the fraction of unemployed workers with unemployment history v who search for type χ jobs, μ_v is the expected suitability of an unemployed worker with unemployment history v, and $\theta^*_{\chi,v}$ is tightness associated with the policy function of unemployed workers with unemployment history v who search for type χ jobs. From (D.1), the measure of unemployed workers who begin the next period unemployed with history $v \in \{2,\ldots,\bar{v}\}$ is given by those who began the previous period unemployed and did not find a job or exit the economy. For $v = 1$, the measure of unemployed workers is simply given by the new entrants to the labor market during stage 3 in the previous period who did not exit in stage 4.

Now let $e_\chi(v,\tau)$ denote the measure of workers with history (v,τ) and are employed at type χ jobs at the beginning of the period. The law of motion for $e_n(v,\tau)$ is given by

$$\hat{e}_n(v,\tau) = \begin{cases} (1-\delta)u(v)\varrho_{n,v}p(\theta^*_{n,v}) & \text{for } \tau = 1, \\ (1-\delta)e_n(v,\tau_-)(1-\lambda\mu_{v,\tau_-}p(\theta^*_{c,v,\tau_-})) & \text{for } \tau \in \{2,\ldots,\bar{\tau}-1\}, \\ (1-\delta)\sum_{\tau=\bar{\tau}}^{\bar{\tau}+1} e_n(v,\tau_-)(1-\lambda\mu_{v,\tau_-}p(\theta^*_{c,v,\tau_-})) & \text{for } \tau = \bar{\tau}, \end{cases} \tag{D.2}$$

where $\mu_{v,\tau}$ is a worker with history (v,τ)'s expected suitability, $\tau_- \equiv \tau - 1$, and $\theta^*_{\chi,v,\tau}$ is tightness associated with the policy function of an employed worker with history (v,τ) in a submarket with type χ jobs. From (D.2), workers who begin the next period employed in non-college jobs and with $\tau = 1$ are comprised of unemployed workers who matched with a non-college job in the previous period. Workers who begin the next period with at least two periods of underemployment history are comprised of those who began the previous period underemployed and did not transition to a college job. All respective measures are multiplied by $(1-\delta)$ as this is the fraction of workers who remain in the labor market across periods.[31] The law of motion for $e_c(v,\tau)$ is given by

$$\hat{e}_c(v,\tau) = \begin{cases} (1-\delta)\left[u(v)\varrho_{c,v}\mu_v p(\theta^*_{c,v}) + e_c(v,\tau) + \phi(\sum_{\tau=1}^{\bar{\tau}}[e_c(v,\tau) + e_n(v,\tau)\lambda\mu_{v,\tau}p(\theta^*_{c,v,\tau})])\right] & \text{for } \tau = 0, \\ (1-\delta)(1-\phi)[e_c(v,\tau) + e_n(v,\tau)\lambda\mu_{v,\tau}p(\theta^*_{c,v,\tau})] & \text{for } \tau \in \{1,\ldots,\bar{\tau}\}. \end{cases} \tag{D.3}$$

The measure of workers who work in college jobs and have zero underemployment experience consists of unemployed workers who find a college job, those workers who are already employed in college jobs with $\tau = 0$, and finally a fraction ϕ of those employed in a college job with $\tau \geqslant 1$ or who transitioned from a non-college to college job and regained their college skills. The measure of workers employed in college jobs with $\tau > 0$ is given by a fraction $1-\phi$ of workers who began the previous period either already employed in a college job or transitioned from a non-college job to a college job and did not regain their college skills.

D.2 Propositions and Proofs

Proposition 1. *Consider a worker with history (v,τ) and expected suitability μ who is currently employed in a type χ job. The worker will never search in a submarket for another type χ job. Moreover, if a worker is employed in a college job, then $\theta^*_{\chi,v,\tau} = 0$ for all $(\chi,v,\tau) \in X \times Y \times T$.*

Proof. Suppose that a worker who is currently employed in a non-college job and searches in a submarket for a non-college job. Their submarket choice is given by

$$\theta = \arg\max\{-k_n\theta + p(\theta)(V_{e,n}(v,\hat{\tau},\mu) - V_{e,n}(v,\hat{\tau},\mu))\}. \tag{D.4}$$

For workers currently employed in college jobs and searching in a submarket for a college job, their choice of tightness is

$$\theta = \arg\max\{-k_c\theta + p(\theta)(V_{e,c}(v,\tau,\mu) - V_{e,c}(v,\tau,\hat{\mu}))\}. \tag{D.5}$$

Clearly the solution to (D.4) and (D.5) is $\theta = 0$. If the worker chose $\theta > 0$ and found another type χ job, then the value of their employment relationship is unchanged from the value of their current employment relationship. Thus, workers employed in a type χ job will never transition to another type χ job.

Now suppose that the worker employed in a college job searches for a non-college job.

[31] We have simplified equations (D.2)-(D.3) by accounting for the fact that workers employed in college jobs will not transition to a non-college job.

Their submarket choice is

$$\theta = \arg\max\{-k_n\theta + p(\theta)(V_{e,n}(v,\tau,\mu) - V_{e,c}(v,\tau,\mu))\}. \tag{D.6}$$

A worker would not transition to a non-college job to only transition back to a college job in the future as underemployment leads to depreciation of college occupation-specific human capital. Therefore, if the worker transitions to a non-college job, they will remain in a non-college job until they exit the labor force. It follows that the sum of the worker's lifetime utility and firm's profits in a non-college job is bounded by

$$\bar{V}_{e,n}(v,\tau,\mu) = \frac{y_n(\bar{\tau})}{1-\beta(1-\delta)}. \tag{D.7}$$

If, however, the worker were to remain employed in the college job until exiting the labor force, the value of their current employment relationship would be given by

$$V_{e,c}(v,\tau,\mu) = \frac{y_c(\tau)}{1-\beta(1-\delta)(1-\phi)} + \frac{\phi\beta(1-\delta)y_c(0)}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\phi)]}. \tag{D.8}$$

Clearly $V_{e,c}(v,\tau,\mu) > \bar{V}_{e,n}(v,\tau,\mu)$ as we have assumed $y_c(\tau) > y_n(\tau)$ for all $\tau \in T$. Therefore, the solution to (D.6) is $\theta = 0$. ∎

Proposition 2. *Assume that $a^H = a^L = 1$, which turns off the unobserved heterogeneity channel. Further, let $\Delta(\tau) = V_{e,c}(\tau) - V_{e,n}(\tau)$. Tightness, θ_τ, satisfies*

$$k_c \geqslant p'(\theta_\tau)\Delta(\tau), \tag{D.9}$$

where $\theta_\tau \geqslant 0$ with complementary slackness. We have the following results:

(i) $\Delta(\tau)$ *is strictly decreasing in* τ.

(ii) $\lambda p(\theta_\tau)$ *is weakly decreasing in* τ.

(iii) $\lambda p(\theta_\tau)$ *is generally concave in* τ.

Proof. We denote $V_{e,\chi}(\tau)$ as the sum of the worker's utility and firm's profits in a match between a type χ job and worker with underemployment history τ. It is straightforward to show:

$$V_{e,n}(\tau) = y_n(\tau) + \beta(1-\delta)\{V_{e,n}(\hat{\tau}) - k_c\hat{\theta} + \lambda p(\hat{\theta})\Delta(\hat{\tau})\}, \tag{D.10}$$

$$V_{e,c}(\tau) = \frac{y_c(\tau)[1-\beta(1-\delta)] + \phi\beta(1-\delta)y_c(0)}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\phi)]}, \tag{D.11}$$

where $\hat{\theta}$ solves

$$k_c \geqslant p'(\hat{\theta})\Delta(\hat{\tau}). \tag{D.12}$$

Part (i): We proceed via proof by contradiction. Suppose that $\Delta(\tau)$ is strictly increasing in τ. Consider $V_{e,n}(\bar{\tau})$ and $V_{e,n}(\bar{\tau}-1)$. It is easy to show that

$$V_{e,n}(\bar{\tau}) - V_{e,n}(\bar{\tau}-1) = y_n(\bar{\tau}) - y_n(\bar{\tau}-1) > 0. \tag{D.13}$$

Now consider

$$\begin{aligned} &V_{e,n}(\bar{\tau}-1) - V_{e,n}(\bar{\tau}-2) = y_n(\bar{\tau}-1) - y_n(\bar{\tau}-2) + \\ &\beta(1-\delta)\{V_{e,n}(\bar{\tau}) - V_{e,n}(\bar{\tau}-1) - k_c\theta^* + \lambda p(\theta^*)\Delta(\bar{\tau}) + k_c\theta^{**} - \lambda p(\theta^{**})\Delta(\bar{\tau}-1)\}, \end{aligned} \tag{D.14}$$

where θ^* and θ^{**}, respectively, solve

$$k_c \geqslant p'(\theta^*)\Delta(\bar{\tau}), \tag{D.15}$$

$$k_c \geqslant p'(\theta^{**})\Delta(\bar{\tau}-1). \tag{D.16}$$

From (D.14), $V_{e,n}(\bar{\tau}-1) - V_{e,n}(\bar{\tau}-2) > 0$ as $y_n(\bar{\tau}-1) > y_n(\bar{\tau}-2)$, $V_{e,n}(\bar{\tau}) > V_{e,n}(\bar{\tau}-1)$ from equation (D.13), and (assuming interior solutions), $-k_c\theta^* + \lambda p(\theta^*)\Delta(\bar{\tau}) > -k_c\theta^{**} + \lambda p(\theta^{**})\Delta(\bar{\tau}-1)$ as $\Delta(\bar{\tau}) > \Delta(\bar{\tau}-1)$ (by assumption) and $\theta^* > \theta^{**}$ following (D.15)-(D.16). We can extend this logic to show that $V_{e,n}(\tau) < V_{e,n}(\hat{\tau})$ for all $\tau \in \{1, 2, \ldots, \bar{\tau}-1\}$ and $\hat{\tau} = \min\{\tau+1, \bar{\tau}\}$. In other words, under the assumption that $\Delta(\tau)$ is increasing in τ, $V_{e,n}(\tau)$ is also increasing in τ. However, we can see from (D.11) that $V_{e,c}(\tau)$ is weakly decreasing in τ as $y_c(\tau)$ is weakly decreasing in τ. Hence, $\Delta(\tau) = V_{e,c}(\tau) - V_{e,n}(\tau)$ is decreasing in τ, which is a contradiction.

Part (ii): We now proceed to show that θ is weakly decreasing in τ. In the main text, we showed that the optimal choice of θ satisfies

$$k_c \geqslant p'(\theta)\Delta(\tau). \tag{D.17}$$

For this part of the proof, we assume an interior solution to (D.17). Following part (i), where we have shown $\Delta(\tau)$ is decreasing in τ, it follows that the optimal θ which satisfies equation (D.17) is decreasing in τ as $p(\theta)$ is strictly concave and, hence, $p'(\theta)$ is strictly decreasing in θ. As $p(\theta)$ is strictly increasing in θ, it follows that $\lambda p(\theta)$ is strictly decreasing in τ for all τ such that $\theta > 0$ satisfies (D.17). If $\theta = 0$ solves (D.17) for some $\tau^* \in T$, it follows that $\theta = 0$ for all $\tau \in \{\tau^*, \ldots, \bar{\tau}\}$ as, following part (i), $\Delta(\tau)$ is decreasing in τ. Hence, $\lambda p(\theta) = 0$ for all $\tau \in \{\tau^*, \ldots, \bar{\tau}\}$ and $\lambda p(\theta)$ is weakly decreasing in τ.

Part (iii): We show that $\lambda p(\theta)$ is generally concave in τ. Assuming an interior solution, θ_τ solves

$$p'(\theta_\tau) = \frac{k_c}{\Delta(\tau)}. \tag{D.18}$$

As $\Delta(\tau)$ is decreasing in τ (shown in part (i)), it follows that $k_c/\Delta(\tau)$ is increasing in τ. From (D.18), θ_τ is decreasing in τ, as $p'(\theta)$ is decreasing in θ. As for the concavity of $p(\theta_\tau)$, it is sufficient to characterize when the function $k_c/\Delta(\tau)$ is convex. To ease the exposition, suppose for the rest of this proof that $\tau \in \mathbb{R}_+$ and that $\Delta(\tau)$ is a twice continuously differentiable function. Let $g(\tau) \equiv [\Delta(\tau)]^{-1}$. The second derivative of $g(\tau)$ is

$$g''(\tau) = \frac{-\Delta''(\tau)[\Delta(\tau)]^2 + 2\Delta'(\tau)\Delta(\tau)\Delta'(\tau)}{[\Delta(\tau)]^4}. \tag{D.19}$$

It follows that $g''(\tau) > 0$, and $k_c/\Delta(\tau)$ is convex, if and only if

$$\frac{2[\Delta'(\tau)]^2}{\Delta(\tau)} > \Delta''(\tau). \tag{D.20}$$

There are three cases to consider.

1. $\Delta''(\tau) = 0$, i.e., $\Delta(\tau)$ is linear. Then (D.20) is satisfied.

2. $\Delta''(\tau) < 0$, i.e., $\Delta(\tau)$ is concave. Then (D.20) is satisfied.

3. $\Delta''(\tau) > 0$, i.e., $\Delta(\tau)$ is convex. In general, (D.20) is not guaranteed to hold. However, as τ increases and $\Delta(\tau)$ approaches zero, the left side of (D.20) approaches infinity. However, as τ increases, the right side of (D.20) decreases. Hence, (D.20) is more likely to be satisfied at higher values of τ in the case where $\Delta''(\tau) > 0$.

To summarize, part (iii) has shown that the function $k_c/\Delta(\tau)$ is generally convex in τ, especially at higher values of τ. It follows that, when $k_c/\Delta(\tau)$ is convex, θ_τ is concave in τ in order to satisfy (D.18) (through the concavity of $p(\cdot)$). If θ_τ is concave, then $p(\theta_\tau)$ is also concave. ∎

Proposition 3. *Consider the worker's expected suitability at underemployment duration, τ, for $\tau \in \{1, 2, \ldots, \bar{\tau}\}$:*

$$\mu_\tau = a^H - \frac{(a^H - \mu_{\tau-1})(1 - pa^L)}{1 - p\mu_{\tau-1}}. \tag{D.21}$$

Suppose that the matching probability of a suitable worker, p, is independent of τ and $p > 0$. We have the following results:

(i) $\mu_\tau = \mu_{\tau-1}$ *if* $\mu_{\tau-1} \in \{a^L, a^H\}$.

(ii) *If* $a^L < \mu_{\tau-1} < a^H$, *then* $\mu_\tau < \mu_{\tau-1}$.

(iii) *If* $\mu_{\tau-1} < 0.5[a^L + a^H]$, *then* $\partial[\mu_\tau - \mu_{\tau-1}]/\partial\mu_{\tau-1} < 0$.

(iv) *Let* $a^L = \alpha a^H$ *where* $\alpha \in [0, 1)$ *and* $\mu_{\tau-1} < a^H$. $\partial[\mu_\tau - \mu_{\tau-1}]/\partial\alpha > 0$.

Proof. Part (i): Substituting $\mu_{\tau-1} = a^L$ into (D.21) gives $\mu_\tau = \mu_{\tau-1} = a^L$. Through the same process, we have $\mu_\tau = \mu_{\tau-1}$ if $\mu_{\tau-1} = a^H$.

Part (ii): Taking the difference between μ_τ and $\mu_{\tau-1}$ gives

$$\mu_\tau - \mu_{\tau-1} = \frac{p(a^H - \mu_{\tau-1})(a^L - \mu_{\tau-1})}{1 - p\mu_{\tau-1}} < 0, \tag{D.22}$$

as $a^L < \mu_{\tau-1} < a^H$. Hence, $\mu_\tau < \mu_{\tau-1}$.

Part (iii): Differentiating (D.22) with respect to $\mu_{\tau-1}$ gives

$$\frac{\partial[\mu_\tau - \mu_{\tau-1}]}{\partial\mu_{\tau-1}} = \frac{p(2\mu_{\tau-1} - a^H - a^L)(1 - p\mu_{\tau-1}) + p^2(a^H - \mu_{\tau-1})(a^L - \mu_{\tau-1})}{(1 - p\mu_{\tau-1})^2}. \tag{D.23}$$

As $a^L < \mu_{\tau-1} < a^H$, it follows that $p^2(a^H - \mu_{\tau-1})(a^L - \mu_{\tau-1}) < 0$. Moreover, $p\mu_{\tau-1} < 1$. Thus, a sufficient condition for $\partial[\mu_\tau - \mu_{\tau-1}]/\partial\mu_{\tau-1} < 0$ is $2\mu_{\tau-1} - a^H - a^L < 0$, or $\mu_{\tau-1} < 0.5 * [a^H + a^L]$.

Part (iv): Replacing a^L with αa^H in equation (D.22) gives

$$\mu_\tau - \mu_{\tau-1} = \frac{p(a^H - \mu_{\tau-1})(\alpha a^H - \mu_{\tau-1})}{1 - p\mu_{\tau-1}}. \tag{D.24}$$

Hence,

$$\frac{\partial[\mu_\tau - \mu_{\tau-1}]}{\partial\alpha} = \frac{p(a^H - \mu_{\tau-1})}{1 - p\mu_{\tau-1}} > 0, \tag{D.25}$$

as $\mu_{\tau-1} < a^H$ and $p\mu_{\tau-1} < 1$. ■

E Model with Full Information

This appendix provides further details on the model with full information referenced in Section 5.2. For brevity, we only present the details in the environment and equilibrium that are new relative to the baseline model presented in Section 3.

E.1 Environment

Workers learn their suitability type upon entering the labor market. A worker's suitability type is public information. The labor market continues to be organized in a continuum of submarkets. In the full information case, however, submarkets are also indexed by the worker's suitability type. Denoting $A = \{L, H\}$ and an individual's worker suitability type by i, the labor market is now organized in a continuum of submarkets indexed by $\omega = (\chi, i, v, \tau, x) \in X \times A \times \Upsilon \times T \times \mathbb{R}$. That is, in submarket ω, type χ firms search for a type-i worker with labor market history (v, τ) and offer suitable workers an employment contract worth x in lifetime utility.

E.2 Value Functions

The value functions in the full information version of the model are very similar to those in the baseline model. The main exception is that a worker's suitability type, i, and probability of being suitable for a college job, a^i, take the place of, μ, the expected suitability in the version with information frictions. Here is the value of an unemployed worker of suitability type i who searches for a non-college job:

$$V_{u,n}(v,i) = b + \beta(1-\delta)\{V_u(\hat{v},i) + R_n(x, V_u(\hat{v},i))\}, \tag{E.1}$$

where

$$V_u(v,i) = \max\{V_{u,n}(v,i), V_{u,c}(v,i)\} \tag{E.2}$$

is the value of unemployment for a type-i worker with unemployment history v and

$$R_\chi(x, V_u(\hat{v},i)) = \max_x p(\theta(\chi, i, \hat{v}, 0, x))(x - V_u(\hat{v},i)). \tag{E.3}$$

The value of searching for a college job satisfies:

$$V_{u,c}(v,i) = b + \beta(1-\delta)\{V_u(\hat{v},i) + a^i R_c(x, V_u(\hat{v},i))\}. \tag{E.4}$$

The sum of the worker's lifetime utility and firm's profits in a match between a non-college job and type-i worker with history (v, τ) is given by:

$$V_{e,n}(v,\tau,i) = y_n(v,\tau) + \beta(1-\delta)\{V_{e,n}(v,\hat{\tau},i) + \lambda a^i S(v,\hat{\tau},i)\}, \tag{E.5}$$

where

$$S(v,\hat{\tau},i) = \max_x p(\theta(c,i,v,\hat{\tau},x))(x - V_{e,n}(v,\hat{\tau},i)). \tag{E.6}$$

Finally, sum of the worker's lifetime utility and the firm's profits in a match between a college job and a type-i worker with history (v,τ), $V_{e,c}(v,\tau,i)$, satisfies

$$V_{e,c}(v,\tau,i) = y_c(v,\tau) + \beta(1-\delta)\{\phi V_{e,c}(1,0,i) + (1-\phi)V_{e,c}(v,\tau,i)\}. \tag{E.7}$$

E.3 Free Entry

In any submarket visited by a positive number of workers, tightness is consistent with the firm's incentives to create vacancies if and only if

$$k_\chi \geqslant q(\theta(\chi,i,v,\tau,x))\{V_{e,\chi}(v,\tau,i) - x\}, \tag{E.8}$$

and $\theta(\chi,i,v,\tau,x) \geqslant 0$ with complementary slackness. We restrict attention to equilibria in which $\theta(\chi,i,v,\tau,x)$ satisfies the complementary slackness condition in every submarket, even those that are not visited by workers.

E.4 Laws of Motion

Let $u_i(v)$ denote the measure of workers of suitability type i who begin the period unemployed with unemployment history v. The law of motion is given by

$$\hat{u}_i(v) = \begin{cases} (1-\delta)\delta\pi^i & \text{for } v = 1, \\ (1-\delta)u_i(v_-)[\varrho_{i,n,v_-}(1-p(\theta^*_{i,n,v_-})) + \varrho_{i,c,v_-}(1-a^i p(\theta^*_{i,c,v_-}))] & \text{for } v \in \{2,\ldots,\bar{v}-1\}, \\ (1-\delta)\sum_{v=\bar{v}}^{\bar{v}+1} u_i(v_-)[\varrho_{i,n,v_-}(1-p(\theta^*_{i,n,v_-})) + \varrho_{i,c,v_-}(1-a^i p(\theta^*_{i,c,v_-}))] & \text{for } v = \bar{v}, \end{cases} \tag{E.9}$$

where $\pi^H = \pi$, $\pi^L = 1-\pi$, $\hat{u}_i(v)$ is the measure of unemployed workers with unemployment history v and suitability type i at the beginning of the next period, $v_- \equiv v-1$, $\varrho_{i,\chi,v} \in [0,1]$ is the fraction of unemployed workers with suitability type i and unemployment history v who search for type χ jobs, and $\theta^*_{i,\chi,v}$ denotes tightness associated with the policy function of unemployed workers with suitability type i and unemployment history v who search for type χ jobs.

Now let $e_{i,\chi}(v,\tau)$ denote the measure of workers with suitability type i and history (v,τ) who are employed at type χ jobs at the beginning of the period. The law of motion for $e_{i,n}(v,\tau)$ is given by

$$\hat{e}_{i,n}(v,\tau) = \begin{cases} (1-\delta)\varrho_{i,n,v}u_i(v)p(\theta^*_{i,n,v}) & \text{for } \tau = 1, \\ (1-\delta)e_{i,n}(v,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,v,\tau_-})) & \text{for } \tau \in \{2,\ldots,\bar{\tau}-1\}, \\ (1-\delta)\sum_{\tau=\bar{\tau}}^{\bar{\tau}+1} e_{i,n}(v,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,v,\tau_-})) & \text{for } \tau = \bar{\tau}, \end{cases} \tag{E.10}$$

where $\tau_- \equiv \tau - 1$, $\theta^*_{i,\chi,v,\tau}$ is tightness associated with the policy function of an employed worker with suitability type i and history (v,τ) in a submarket with type χ jobs.

The law of motion for $e_{i,c}(v,\tau)$ is given by

$$\hat{e}_{i,c}(v,\tau) = \begin{cases} (1-\delta)[e_c(v,\tau) + u_i(v)\varrho_{i,c,v}a^i p(\theta^*_{i,c,v}) + \phi(e_{i,c} - e_{i,c}(1,0) + e^*_{i,n} + u^*_{i,c})] & \text{for } v = 1 \text{ and } \tau = 0, \\ (1-\delta)(1-\phi)[u_i(v)\varrho_{i,c,v}a^i p(\theta^*_{i,c,v}) + e_{i,c}(v,\tau)] & \text{for } v \geqslant 2 \text{ and } \tau = 0, \\ (1-\delta)(1-\phi)[e_{i,n}(v,\tau)\lambda a^i p(\theta^*_{i,c,v,\tau}) + e_{i,c}(v,\tau)] & \text{for } v \geqslant 2 \text{ and } \tau \geqslant 1, \end{cases} \tag{E.11}$$

where $e_{i,c} = \sum_{v\in\Upsilon}\sum_{\tau\in T} e_{i,c}(v,\tau)$ is the total measure of type-i workers employed in college jobs at the beginning of a period, $e^*_{i,n} = \lambda\sum_{v\in\Upsilon}\sum_{\tau\in T} e_{i,n}(v,\tau)a^i p(\theta^*_{i,c,v,\tau})$ is the total measure of type-i workers who transitioned from a non-college to college job within the period, and $u^*_{i,c} = a^i\sum_{v=2}^{\bar{v}} u_i(v)\varrho_{i,c,v}p(\theta^*_{i,c,v})$ is the total measure of unemployed workers with unemployment history $v \in \{2,\ldots,\bar{v}\}$ who found a college job in the previous period.

E.5 Equilibrium Definition

Definition 2. A stationary recursive equilibrium consists of a market tightness function $\theta(\omega): X \times A \times \Upsilon \times T \times \mathbb{R} \to \mathbb{R}_+$, a value function for unemployed workers, $V_u(v,i): \Upsilon \times A \to \mathbb{R}$, a policy function for unemployed workers, $\omega^*_u(v,i): \Upsilon \times A \to X \times \mathbb{R}$, a joint value function for the worker-firm match, $V_{e,\chi}(v,\tau,i): X \times \Upsilon \times T \times A \to \mathbb{R}$, a policy function for the worker-firm match, $\omega^*_{e,\chi}(v,\tau,i): X \times \Upsilon \times T \times A \to X \times \mathbb{R}$, and a distribution of workers across the states of employment. The functions satisfy the following conditions. First, $\theta(\omega)$ satisfies (E.8) and the slackness condition for all $\omega \in X \times A \times \Upsilon \times T \times \mathbb{R}$. Third, $V_u(v,i)$ satisfies (E.2) for all $(v,i) \in \Upsilon \times A$ and $\omega^*_u(v,i)$ is the associated policy function. Fourth, $V_{e,n}(v,\tau,i)$ and $V_{e,c}(v,\tau,i)$ satisfy equations (E.5) and (E.7) for all $(v,\tau,i) \in \Upsilon \times T \times A$ and $\omega^*_{e,\chi}(v,\tau,i)$ is the associated policy function. Finally, the distribution of workers satisfies the laws of motion specified in Section E.4.

E.6 Quantitative Analysis

To take a closer look at the role of information friction in determining duration dependence, we assume the worker's suitability type is publicly observable. The first result that emerges from removing information frictions is that broad-suitable workers ($i = H$) never search for non-college jobs, as illustrated by the unemployed worker's policy function in Figure E.1. Consequently, the pool of underemployed workers consists solely of limited-suitability workers ($i = L$).

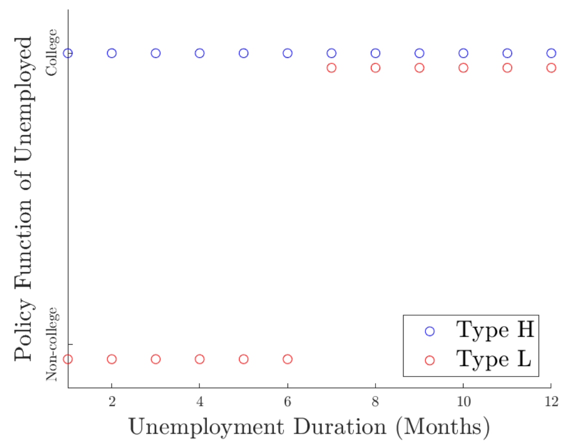
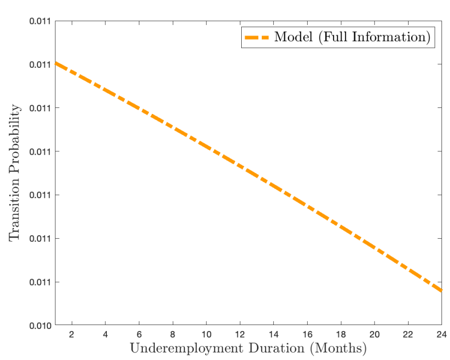
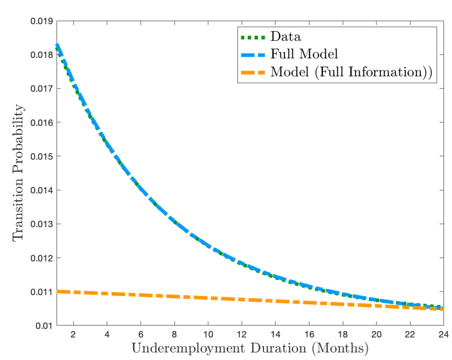


Figure E.1: Policy Function of the Unemployed

Figure E.2 shows that, with the pool of underemployed workers being exclusively composed of limited-suitability workers, a mild negative duration dependence is still observed. Notably, the transition probability decreases from 0.01100 at $\tau = 1$ to 0.01048 at $\tau = 24$. The magnitude of this decline is negligible when compared to the full model.



(a) No Information Frictions



(b) Model and Data Comparisons

Figure E.2: Duration Dependence with and without Information Frictions